(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAP 223**

COURSE TITLE: **ALGEBRAIC STRUCTURES II**

DATE: 8/10/2021 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30MKS)

a) Define the following terms;

I) Binary relation (2mks)

II) Inverse relation (2mks)

III) Limit of a sequence (2mks)

IV) Null sequence (2mks)

V) Bounded sequence (1mk)

VI) A ring (3mks)

VII) A field (3mks)

VIII) Complementary relation (2mks)

b) Let R C X x Y be a binary relation from X to Y. Let A, B C X be the subsets. Prove that the following statements are true.

I) If A C B, then R (A) C R (B) (3mks)

II) R (AUB) = R (A) U R (B) (3mks)

III) R (A∩B) C R (A)∩R(B) (3mks)

C) Let R_1, R_2 C X x Y be relations from X to Y. If $R_1(x) = R_2(x)$ for all x that belongs to X, show that $R_1 = R_2$ (4mks)

QUESTION TWO (20MKS)

2. a) Prove that the sum of the first n natural numbers is given by the formula:

$1+2+3+\ldots\ldots\ldots+n=\frac{n(n+1)}{2}$ (5mks)

b) Show that a composite integer n has a prime factor less than or equal to $\sqrt{n}$ (5mks)

c) Prove that every convergent sequence is bounded (5mks)

d) Prove that $n^3 + 2n$ is divisible by 3 for all n ϵ (5mks)

QUESTION THREE (20MKS)

a i) State 3 properties of a ring (3mks)

ii) In a field a product of two non – zero elements is non zero, prove (2mks)

iii) Prove that for any elements a, b and c of a field and non- zero element the cancellation law ab=ac implies b=c holds (2mks)

b i) define the following terms a group state 3 conditions that a group satisfies (4mks)

ii) Order of a group (2mks)

iii) Abelian group and give two examples (3mks)

c) Let (a,*) be a group and let a, b, c belong to G, prove that the following hold (4mks)

i) a * b = a * c implies b=c

ii) b * a = c * a implies b=c

QUESTION FOUR (20MKS)

a) i) Let G={$a_1, a_2 \ldots\ldots a_n$} be a finite group prove that in any row and column of the multiplication table of G , each element of G appears once. (3mks)

ii) Let G be a group a belongs to G. Prove that <a> is a subgroup of G . (5mks)

iii) Define a subgroup (2mks)

b) Let G be a group, $H \leq G$ and $K \leq G$ prove that $H \cap K \leq G$ (10 mks)

QUESTION FIVE (20mks)

(a) Let f: $G \rightarrow H$ be a homomorphism of groups. Denote the identity of G by e_G and the identity of H by e_H. Show that f

(b) Preserves identities: $f(e_G) = e_H$ (2mks)

(c) Preserves inverses: for every $x \in G$, $f(x^{-1}) = f(x)$ (8mks)

(d) The center z of a group G is defined by z= {$z \in G$ |zx=xz for all $x \in G$}, prove that z is a subgroup of G. (4mks)

(e) Let G be the group of all non-zero complex numbers a+ib, (a,b real but not both zero) under multiplication and let
H= {$a+ib \in G \mid a^2 + b^2 = 1$} verify that H is a subgroup of G. (6mks).