(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR
FOURTH YEAR SECOND SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: STA 442

COURSE TITLE: MULTIVARIATE ANALYSIS

DATE: 5/10/2021 **TIME:** 9:00 AM – 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer **Questions ONE** and **ANY OTHER TWO**.

QUESTION ONE (30 MARKS)

(a) (i) What is a mean vector [1 mark]

(ii) Describe how multivariate data are arranged [3 marks]

(b) The data below shows the scores of a sample of 15 students in mathematics, English and Kiswahili CATS in a certain school

$$\boldsymbol{X} = \begin{bmatrix} 4 & 8 & 6 & 8 & 9 \\ 8 & 7 & 4 & 4 & 10 \\ 10 & 9 & 7 & 5 & 9 \end{bmatrix}$$

Obtain

(i) Mean Vector [3 marks]

(ii) Variance-Covariance matrix [5 marks]

(iii)Correlation matrix [3 marks]

(c) Let $\underline{x} = [5,1,3]$ and $\underline{y} = [-1,3,1]$. Find

(i) The length of $\underline{x}$ [2marks]

(ii) The angle between $\underline{x}$ and $\underline{y}$ [3marks]

(iii) The length of the projection of $\underline{x}$ on $\underline{y}$ [2mark]

(d) A random sample of 10 was obtained from a bivariate normal population with mean vector $\boldsymbol{\mu}$ and a known variance-covariance matrix $\Sigma_0 = \begin{bmatrix} 4 & 4.2 \\ 4.2 & 9 \end{bmatrix}$. Find the principal component and hence Test at $\alpha = 0.01$ level of significance for $H_0\text{:}\,\boldsymbol{\mu} = \boldsymbol{\mu}$ vs $H_1\text{:}\,\boldsymbol{\mu} \neq \boldsymbol{\mu_0}$ where $\boldsymbol{\mu_0} = (6,5)'$ and the sample mean vector is $\underline{\overline{X}} = (5.8, 5.2)'$ [8 marks]

QUESTION TWO (20MARKS)

(a) Let $\underline{x}$ be a p-variate random vector with mean vector $\underline{\mu}$ and variance covariance matrixΣ, show that$E(\underline{XX'}) = \Sigma + \underline{\mu\mu}'$, hence show that $E(\underline{X}'A\underline{X}) = trace(A\Sigma) + \underline{\mu}'A\underline{\mu}$ where A is a symmetric matrix of constants. (8mks)

(b) Find the symmetric matrix A for a quadratic form $Q(X_1, X_2, X_3) = 9X_1^2 + 16X_1X_2 + X_2^2 + 8X_1X_3 + 6X_2X_3 + 3X_3^2$. Hence obtain the expected value of $Q(X_1, X_2, X_3)$ and $E(X'AX)$ given that $\underline{\boldsymbol{\mu}} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & -2 & 10 \\ -2 & 4 & 3 \\ 10 & 3 & 9 \end{pmatrix}$ (12mks)

QUESTION THREE (20 MARKS)

(a) Let $X_1 \sim N_2(\mu_1, \Sigma)$ and $X_2 \sim N_2(\mu_2, \Sigma)$. Independent random samples of size 10 and 9 were taken from X_1 and X_2 respectively. The summary statistics are as follows:

$$\underline{\overline{X}}_1 = \begin{bmatrix} 55 \\ 34 \end{bmatrix}, \underline{\overline{X}}_2 = \begin{bmatrix} 60 \\ 43 \end{bmatrix}, S_1 = \begin{bmatrix} 20 & 10 \\ 10 & 10 \end{bmatrix}, S_2 = \begin{bmatrix} 4 & 16 \\ 16 & 9 \end{bmatrix}$$

(i) Obtain the pooled sample variance-covariance matrix S_p [3marks]

(ii) Test the hypothesis at $\alpha = 0.05$ level of significance

$$H_0: \boldsymbol{\mu} = \boldsymbol{\mu_0}$$
$$H_0: \boldsymbol{\mu} = \boldsymbol{\mu_1}$$

Where Σ is unknown [5marks]

(b) For a bivariate normal distribution, use the data below to test at $\alpha = 0.05$ level the hypothesis

$$\begin{matrix} H_0: \boldsymbol{\mu} = (3.4, 6)' \\ H_1: \boldsymbol{\mu} = (3.4, 6)' \end{matrix} \; Vs$$

$$\underline{X} = \begin{bmatrix} 3 & 4 & 5 & 6 & 2 \\ 9 & 5 & 7 & 2 & 8 \end{bmatrix}$$ [7 marks]

(c) Let $\underline{x}$ be a random vector having the covariance matrix

$$\Sigma = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$$

Obtain

(i) Square root of $\Sigma = \left(V^{\frac{1}{2}}\right)$ (2mk)

(ii) Inverse of the square root $\Sigma = \left(V^{\frac{1}{2}}\right)^{-1}$ (1mk)

(iii) Correlation matrix ρ defined by

$$\rho = \left(V^{\frac{1}{2}}\right)^{-1} \Sigma \left(V^{\frac{1}{2}}\right)^{-1}$$ (2mks)

QUESTION FOUR (20 MARKS)

a) Let $\underline{x}$ be a trivariate random vector such that

$E(\underline{x}) = 0$ and $var(\underline{x}) = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix}$. Find the expected value of the quadratic form

$Q = (x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_1)^2$ (5mks)

b) Using the variance-covariance matrix in part (g) find the variance covariance matrix of $Y = (Y_1, Y_2)$ where $Y_1 = x_1 + x_2$ and

$Y_2 = x_1 + 2x_2 + x_3$ (5mks)

(a) Let the multivariate normal distribution be given by

$$f(y_1, y_2, y_3) = \begin{cases} K \exp \frac{-1}{2}[q] \\ 0, \quad otherwise \end{cases}$$

where K is a constant and

$$q = \left\{ 3y_1^2 + 2y_2^2 + 4y_3^2 - 4y_1y_2 + 8y_1y_3 - 6y_2y_3 + 12y_1 + 10y_2 + y_3 \right\}$$

Find the variance-covariance matrix Σ and the mean vector $\underset{\sim}{\mu}$. (10mks)

QUESTION FIVE (20 MARKS)

Observations on three responses are collected from two treatments as shown in the table below

Response \ Treatment	1	1	1	1	2	2
Y_1	7	8	9	6	7	5
Y_2	12	15	13	10	12	10
Y_3	6	6	7	5	7	5

Obtain

(i) Between Treatment sum of squares [6 marks]

(ii) Within treatment sum of squares [5 marks]

(iii)MANOVA table [2 marks]

(iv)Test at $\alpha = 0.05$level of significance that there is no treatment effect. [7 marks]