(Knowledge for Development)

# KIBABII UNIVERSITY

## UNIVERSITY EXAMINATIONS

## 2020/2021 ACADEMIC YEAR

## FIRST YEAR SECOND SEMESTER

## SPECIAL/SUPPLEMENTARY EXAMINATION

## FOR THE DEGREE OF BACHELOR OF EDUATION AND BACHELOR OF SCIENCE

**COURSE CODE:** **MAA 123**

**COURSE TITLE:** CALCULUS II

**DATE:** 29/09/2021 **TIME:** 11:00 AM- 1:00 PM

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## INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

## QUESTION ONE

**(30 MARKS)**

(a)(i) Show that $\int vdu = vu - \int udv$ (2 marks)

(ii) Hence or otherwise show that $\int x\tan^{-1}xdx = \frac{x^2}{2}\tan^{-1}x + \frac{\tan^{-1}x}{2} - \frac{x}{2} + c_x$ (5 marks)

(b) (i) Express $\frac{tanx}{1-3tan^2x}$ in patial fractions (5 marks)

(ii) Use partial fractions to find the integral $\int \left(\frac{2x+1}{(x^2-1)\,(x^2+2)}\right)dx$ (5 marks)

(c) By sketching the graph, find the volume generated when a curve bounded by $y = x^3$ and the $y - axis$ is revolved through four right angles about the $y$ axis where $1 \le x \le 2$ (3 marks)

(d) (i) Express $\tan 3x$ in terms of $\tan x$ (2 marks)

(ii) Determine $\int tan^3xdx$ (2 marks)

(iii) Find $\int sin^2x\, cos^3x\ dy)$ (2 marks)

(e) Use the reduction formula for $I_n = \int_0^{\pi/2} cos^n xdx$, to show that

$$I_{(n-4)} = \left(\frac{n-5}{n-4}\right) I_{(n-6)} \qquad (n \ge 2)$$ (4 marks)

## QUESTION TWO

**(20 marks)**

(a) Find (i) $\int x^2 sin^{-1}x\, dx$ (2 marks)

(ii) $\int \frac{1}{\sqrt{3-2x^2}}\, dx$ (2 marks)

(iii) $\int_1^{\sqrt{3}} \left(\frac{2x}{1+x^2}\right) dx$ (2 marks)

(b) Find the area enclosed by the curve $y = \frac{1}{X+2}$ *and*

(i) the lines $x = 3, x = 5$ (1 marks)

(20 marks)

## QUESTION FOUR

(a) $\int \left(x^{-3/4} + sin^{-3}x\right) dx$ (7 marks)

(b) Find the length of the curve given by $r = a(1 + cos\theta)$ (5 marks)

(c) Show that the volume of cone is $\frac{4}{3}\pi r^2 h$. (4 marks)

(d) Show that for the area bounded by the radius r and a curve $r = f(\theta)$ $is$ $A = \frac{1}{2} r^2(\beta - \propto)$ (4marks)

Where $\propto \leq \theta \leq \beta$

(20 marks)

## QUESTION FIVE

The Equation of a curve in polar co-ordinates is $r = 2(sin\theta + \cos\theta)$

(a) (i) Copy and complete the table for the domain $0 \leq \theta \leq \pi$ (2marks)

| $\theta$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | $\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $2\,sin\,\theta$ | 0 | 1 | 1.41 | – | – | – | – | 1 | – |
| $2\,cos\,\theta$ | 2 | 1.73 | 1.41 | – | – | – | – | – 1.73 | – |

(ii) Plot the curve for the domain $0 \leq \theta \leq \pi$ (2 marks)

(b) (i) Show that the area of sector of the curve $r = 2(\sin\theta + \cos\theta)$ $bounded$ $by$ $\theta = a$ and $\theta = b$ where $0 \leq \leq a < b \leq \pi$ is given by $\int_a^b 2\,(1 + \sin 2\theta)d\theta$ (3marks)

(ii)Determine the area of the sector of this curve bounded by $\theta = 0$ $and$ $\theta = \pi/2$ (2marks)

(iii)Determine the area bounded by the whole curve (2marks)

(c)Given that $I_n = \int cos^n\, x\, dx$, use Integration by parts to show that

$n\, I_n = \cos^{n-1} x \sin x + (n-1)I_{n-2}$, Hence evaluate $\int_0^{\pi/2} cos^5\ x\, dx.$ (9 marks)