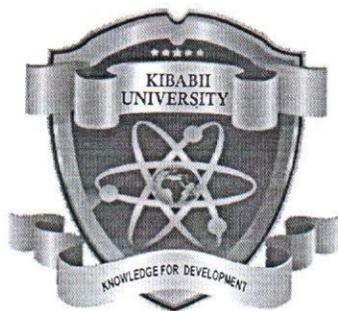


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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

**FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE**

COURSE CODE: MAA 123

COURSE TITLE: CALCULUS II

DATE: 29/09/2021

TIME: 11:00 AM- 1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

(30 MARKS)

QUESTION ONE

(2 marks)

(a)(i) Show that $\int vdu = vu - \int udv$

(ii) Hence or otherwise show that $\int x \tan^{-1} x dx = \frac{x^2}{2} \tan^{-1} x + \frac{\tan^{-1} x}{2} - \frac{x}{2} + c_x$ (5 marks)

(5 marks)

(b) (i) Express $\frac{\tan x}{1-3\tan^2 x}$ in partial fractions

(ii) Use partial fractions to find the integral $\int \left(\frac{2x+1}{(x^2-1)(x^2+2)} \right) dx$ (5 marks)

(c) By sketching the graph, find the volume generated when a curve bounded by $y = x^3$ and the y -axis is revolved through four right angles about the y axis where $1 \leq x \leq 2$ (3 marks)

(2 marks)

(d) (i) Express $\tan 3x$ in terms of $\tan x$

(2 marks)

(ii) Determine $\int \tan^3 x dx$

(2 marks)

(iii) Find $\int \sin^2 x \cos^3 x dy$

(e) Use the reduction formula for $I_n = \int_0^{\pi/2} \cos^n x dx$, to show that

(4 marks)

$$I_{(n-4)} = \left(\frac{n-5}{n-4} \right) I_{(n-6)} \quad (n \geq 2)$$

(20 marks)

QUESTION TWO

(a) Find (i) $\int x^2 \sin^{-1} x dx$

(2 marks)

(ii) $\int \frac{1}{\sqrt{3-2x^2}} dx$

(2 marks)

(iii) $\int_1^{\sqrt{3}} \left(\frac{2x}{1+x^2} \right) dx$

(2 marks)

(b) Find the area enclosed by the curve $y = \frac{1}{x+2}$ and

(1 marks)

(i) the lines $x = 3, x = 5$

QUESTION FOUR

(20 marks)

(a) $\int (x^{-3/4} + \sin^{-3}x) dx$

(7 marks)

(b) Find the length of the curve given by $r = a(1 + \cos\theta)$

(5 marks)

(c) Show that the volume of cone is $\frac{4}{3}\pi r^2 h$.

(4 marks)

(d) Show that for the area bounded by the radius r and a curve $r = f(\theta)$ is $A = \frac{1}{2} r^2 (\beta - \alpha)$

(4marks)

Where $\alpha \leq \theta \leq \beta$

(20 marks)

QUESTION FIVE

The Equation of a curve in polar co-ordinates is $r = 2(\sin\theta + \cos\theta)$

(a) (i) Copy and complete the table for the domain $0 \leq \theta \leq \pi$

| | | | | | | | | | |
|-----------------|---|-----------------|-----------------|-----------------|-----------------|------------------|------------------|------------------|-------|
| θ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ | π |
| $2 \sin \theta$ | 0 | 1 | 1.41 | - | - | - | - | - | 1 |
| $2 \cos \theta$ | 2 | 1.73 | 1.41 | - | - | - | - | - | 1 |

(2marks)

(2 marks)

(ii) Plot the curve for the domain $0 \leq \theta \leq \pi$

(b) (i) Show that the area of sector of the curve $r = 2(\sin\theta + \cos\theta)$ bounded by $\theta = a$ and $\theta = b$ where $0 \leq a < b \leq \pi$ is given by $\int_a^b 2(1 + \sin 2\theta) d\theta$ (3marks)

(ii) Determine the area of the sector of this curve bounded by $\theta = 0$ and $\theta = \pi/2$ (2marks)

(2marks)

(iii) Determine the area bounded by the whole curve

(c) Given that $I_n = \int \cos^n x dx$, use Integration by parts to show that

$$n I_n = \cos^{n-1} x \sin x + (n-1) I_{n-2}, \text{ Hence evaluate } \int_0^{\pi/2} \cos^5 x dx. \quad (9 \text{ marks})$$