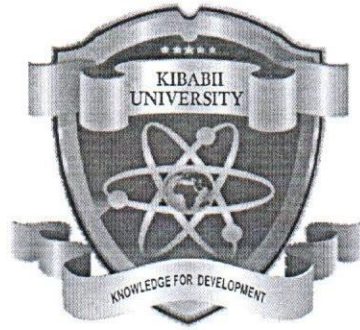


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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER
MAIN EXAMINATION
FOR THE DEGREE OF MASTER OF SCIENCE IN
PURE MATHEMATICS

COURSE CODE: MAT 827
COURSE TITLE: CODING THEORY
DATE: 7/10/21 **TIME:** 9 AM -12 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

MAT 830: CODING THEORY EXAM

August 2021

Answer any Three questions neatly and precisely.
Time: 3 hrs. (Each question carries 20 marks)

Question One

- (a) Define what is meant by
- a q -ary code C of length n . (2mks)
 - The distance $d(x, y)$ between two elements x and y of C (2mks)
 - The minimum distance $d(C)$ of C (2 mks)
- (b) A *bsc* has probability $p = 0.05$ of incorrect transmission, If the codeword 011011101 is transmitted, what is the probability that we receive
- $r = 011111101$ i.e one error (2 mks)
 - $r = 111011100$ (2mks)
 - a double error occurs (3mks)
- (c) State the Sphere packing bound for a q -ary $(n, M, 2e + 1)$ code (3 mks)
- (d) show that for a q -ary $(q + 1, M, 3)$ code, $M = q^{q-1}$ (4mks)

Question Two

For a Linear $[n, k]$ code, Define what is meant by

- a generator matrix (2mks)
 - a Parity check matrix (2 mks)
- (b) Let H be a parity check matrix for the binary Linear code C where

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

- Find $n, k, |C|$ (4 mks)
 - Reduce H to standard form and find a generator matrix for C (4mks)
 - What are $d = d(C)$ and $d^\perp = d(C^\perp)$ explaining your answer (4mks)
 - For the code C find the coset leaders and their syndromes using the given matrix H (4mks)
 - How many received messages can be correctly decoded? Given a reason (2mks)
- Decode the following received vectors $y = 111010$ and $y' = 100011$ (2mks)

Question Three

- (a) Describe the construction of a q -ary Hamming code. State n , k , d for this code (6mks)
(b) Construct a parity check matrix for $\text{Ham}(4, 2)$ (4mks)
(c) state a criterion for finding the minimum distance d of an $[n, k]$ code from a parity check matrix (4mks)
(d) Define what is meant by MDS code (2mks)
(e) Let C and D be two codes over F_3 with generator matrices

$$g = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

and

$$H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Prove that C and D are equivalent codes (4mks)

Question Four

- (a) (i) Define what is meant by a cyclic code (2mks)
(ii) State the necessary and sufficient condition for a code C in $R_n = F_q[x]/x^n - 1$ to be cyclic. (3mks)
(iii) Define what is meant by the generator polynomial of a cyclic code in R_n (2mks)
(b) Factorize $X^6 + 1$ over F_2 and deduce the total number of cyclic codes of length 6 over this field. (6mks)
(c) Write out the generator polynomials and generator matrices for codes in (b) that have dimension 5. In each case deduce the minimum weight of the non-zero codeword. (7 mks)

Question Five

- (a) (i) Construct the field $GF(2^3)$ using $h(x) = 1 + x^3 + x^4$ (4mks)
(ii) Find the minimum polynomial of $\alpha = \beta^3, \alpha \in GF(2^4)$ Constructed above (4mks)
(b)(i) Find the generator polynomial for a cyclic code of length 15 having roots $1, \beta^7, \beta^5 \in GF(2^4)$. (4mks)
(ii) Construct a parity check matrix for this code (4mks)
(c) Construct a triple error correcting BCH code with block length $n = 31$ over $GF(2^5)$ (4mks)