(13)

(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

COURSE CODE: **MAT 827**

COURSE TITLE: CODING THEORY

DATE: 7/10/21 **TIME:** 9 AM -12 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

MAT 830: CODING THEORY EXAM

August 2021

Answer any Three questions neately and precisely.
Time: 3 hrs. (Each question carries 20 marks)

Question One

(a)Define what is meant by
(i) a $q-$ary code C of length n. (2mks)
(ii) The distance $d(x, y)$ between two elements x and y of C (2mks)
(iii) The minimum distance $d(C)$ of C (2 mks)
(b) A *bsc* has probability $p = 0.05$ of incorrect transmission, If the codeword 011011101 is transmitted, what is the probability that we receive
(i)$r = 011111101$ i.e one error(2 mks)
(ii) $r = 111011100$ (2mks)
(iii) a double error occurs (3mks)
(c) State the Sphere packing bound for a $q-$ary $(n, M, 2e + 1)$ code (3 mks)
(d) show that for a $q-$ary $(q + 1, M, 3)$ code, $M = q^{q-1}$ (4mks)

Question Two

For a Linear $[n, k]$ code, Define what is meant by
(i) a generator matrix (2mks)
(ii) a Parity check matrix (2 mks)
(b) Let H be a parity check matrix for the binary Linear code C where

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(i) Find n, k, $|C|$ (4 mks)
(ii) Reduce H to standard form and find a generator matrix for C (4mks)
(iii) What are $d = d(C)$ and $d^{\perp} = d(C^{\perp})$ explaining your answer (4mks)
(iv) For the code C find the coset leaders and their syndromes using the given matrix H (4mks)
(v) How many received messages can be correctly decoded? Given a reason (2mks)
Decode the following received vectors $y = 111010$ and $y' = 100011$ (2mks)

Question Three

(a) Describe the construction of a $q-$ary Hamming code. State n, k, d for this code (6mks)
(b) Construct a parity check matrix for Ham$(4, 2)$ (4mks)
(c) state a criterion for finding the minimum distance d of an $[n, k]$ code from a parity check matrix (4mks)
(d) Define what is meant by MDS code (2mks)
((e) Let C and D be two codes over F_3 with generator matrices

$$g = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

and

$$H = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Prove that C and D are equivalent codes (4mks)

Question Four

(a) (i) Define what is meant by a cyclic code (2mks)
(ii) State the necessary and sufficient condition for a code C in $R_n = F_q[x]/x^n - 1$ to be cyclic. (3mks)
(iii) Define what is meant by the generator polynomial of a cyclic code in R_n (2mks)
(b) Factorize $X^6 + 1$ over F_2 and deduce the total number of cyclic codes of length 6 over this field. (6mks)
(c)Write out the generator polynomials and generator matrices for codes in (b) that have dimension 5. In each case deduce the minimum weight of the non-zero codeword. (7 mks)

Question Five

(a) (i) Construct the field $GF(2^3)$ using $h(x) = 1 + x^3 + x^4$ (4mks)
(ii) Find the minimum polynomial of $\alpha = \beta^3, \alpha \in GF(2^4)$ Constructed above (4mks)
(b)(i)Find the generator polynomial for a cyclic code of length 15 having roots $1, \beta^7, \beta^5 \in GF(2^4)$. (4mks)
(ii)Construct a parity check matrix for this code (4mks)
(c) Construct a tri ple error correcting BCH code with block length $n = 31$ over$GF2^5$ (4mks)