(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAA 225/MAT304**

COURSE TITLE: **COMPLEX ANALYSIS I**

DATE: 10/02/2021 **TIME: 8 AM -10 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE: COMPULSORY (30 MARKS)

a. Given $f(z) = u + iv$ is analytic in a region $\mathbb{R}$. Prove that u and v are harmonic in $\mathbb{R}$ if they have continuous second partial derivatives in $\mathbb{R}$. (5 marks)

b. Let $f(z) = \ln(1 + z)$, where we consider the branch that has the zero value when $z = 0$, expand $f(z)$ in a Taylor series about $z = 0$. Hence expand $\ln(1 + z/1 - z)$ in a Taylor series about $z = 0$ (6marks)

c. Given that $f(z)=u+iv$ is an analytic function and suppose $u(x,y) = e^x(x\sin y - y\cos y)$, find $v(x,y)$ (9 marks)

d. State and prove the Residue Theorem (5marks)

e. Using Cauchy's integral formula, evaluate $\int_c \frac{2z^2+z}{z^2-1}dz$ where C is $|z-1|=1$ (5 marks)

QUESTION TWO (20 MARKS)

a. Determine if the function $U(x,y) = e^x(y\cos 2y + x\sin 2y)$ is harmonic. (5 marks)

b. Show that $f'(z)$ for $f(z)=\bar{z}$ does not exist (5 marks)

c. Verify the Cauchy- Riemann equations for $f(z)=e^z$ (5 marks)

d. Evaluate $\int_{1+i}^{2+3i}(z^2+z)dz$ along the line joining the points (1, -1) and (2, 3) (5marks)

QUESTION THREE (20 MARKS)

a. Given that $f(z)=u+iv$ is an analytic function and suppose that $u=x^2+4x-y^2+2y$, use the Cauchy-Riemann equations to determine the imaginary part v (6marks)

b. Find the residues of $f(z)=\frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane and hence evaluate $\oint_c f(z)dz$ (9marks)

c. Show that if $f(z)$ is analytic and $f'(z)$ is continuous at every point inside and on a simple closed curve C then $\int_C f(z)dz = 0$ (5marks)

QUESTION FOUR (20 MARKS)

a) If $f(z)$ is analytic within and on simple closed curve C and if a is any point within C, show that $f(a) = \frac{1}{2\pi i}\oint_c \frac{f(z)}{z-a}dz$ (5marks)

b) Prove that $\lim_{z\to i} \frac{3z^4-2z^3+8z^2-2z+5}{z-i} = 4+4i$ (5 marks)

c) Using the definition of derivative, find the derivative of $w = f(z) = z^2 - 2z + 1$ at $z = z_o$ and $z = -1$ (5 marks)

d) Is the function $f(z) = \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$ continuous at $z = i$? (5 marks)

QUESTION FIVE

a) Evaluate $\int \bar{z}dz$ from $z = 0$ to $z = 4+2i$ along the curve C given by $z = t^2 + it$

(7 marks)

b) State the De Moivre's theorem and use it to solve $Z^5 = 3 - 4i$ (5 marks)

c) Evaluate $(-1+i)^{1/3}$ and represent first three solutions graphically (8 marks)