(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY

COURSE CODE: **MAT 121**

COURSE TITLE: LINEAR ALGEBRA I

DATE: 12/7/2021 **TIME:** 9 AM - 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Find all values of a for which $X = (a^2 - a, -3, -1)$ and $Y = (2, a - 1, 2a)$ are orthogonal. (5marks)

b) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \\ -1 & 3 & 1 \end{bmatrix}$

i. Compute det (A) using a cofactor expansion along row 2. (4marks)

ii. Find $(A\text{adj}(A))$. (4marks)

iii. Find det $(2A^{-1}\text{adj}(A))$. (5marks)

c) Let **u** and **v** be two solutions of the homogenous system $A\mathbf{x} = O$. Show that $r\mathbf{u} + s\mathbf{v}$ (for , $s \in \mathbb{R}$) is a solution to the same system. (4marks)

d) Show that for any $A,B,C \in M_{n\times n}$, if $AB = AC$ and $|A| \neq 0$, then $B = C$. (4marks)

e) Find a vector X, of length 6, in the opposite direction of $Y = (1, 2,-2)$. (4marks)

QUESTION TWO (20 MARKS)

a) Let $A = \begin{bmatrix} 1 \\ 3 \\ -1 \\ 3 \end{bmatrix}$. Find all constants $c \in \mathbb{R}$ such that $(cA)^T \cdot (cA) = 5$. (5marks)

b) Solve the following system using the Guass-Jordan method. (6marks)

$x_1 + x_2 - x_3 + 4x_4 = 1$

$x_2 - 3x_3 + 4x_4 = 0$

$2x_1 + 2x_2 - 2x_3 + 8x_4 = 2$

c) Let **u** and **v** be two solutions of the non-homogenous system $A\mathbf{x} = B$. Show that $\mathbf{u} - \mathbf{v}$ is a solution to the homogenous system $A\mathbf{x} = O$. (4marks)

d) Verify that the triangle with vertices $A(1, 1, 2)$, $B(1, 2, 3)$, and $C(3, 0, 3)$ is a right triangle. (5marks)

QUESTION THREE (20 MARKS)

f) Let $x, y \in \mathbb{R}^n$ with $\| x \| = \| y \|$. Show that $\mathbf{x} - \mathbf{y}$ and $\mathbf{x} + \mathbf{y}$ are orthogonal. (5marks)

g) Solve the following system AX=B. $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ (6marks)

h) Show that if **x** and **y** are in $\mathbb{R}^n$, then $\| x + y \| \leq \| x \| + \| y \|$ (4marks)

i) Let A be a square matrix. Show that if $A = 2A^T$, then $A = O$. (5marks)

QUESTION FOUR (20 MARKS)

a) Let $detA = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$=7, evaluate $\begin{vmatrix} b_1 & b_2 & b_1 - 3b_3 \\ a_1 & a_2 & a_1 - 3a_3 \\ c_1 & c_2 & c_1 - 3c_3 \end{vmatrix}$ (5marks)

b) Show that $U \cdot (V + W) = U \cdot V + U \cdot W$, for any vectors $U, V, W \in \mathbb{R}^n$. (4marks)

c) Let $A = \begin{bmatrix} a+e & b+f \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $C = \begin{bmatrix} e & f \\ c & d \end{bmatrix}$. Show that |A|=|B|+|C|

(5marks)

d) Solve the following linear system (6marks)

$x + 3y - z + w = 1$

$2x - y - 2z + 2w = 2$

$3x + y - z + w = 1$

QUESTION FIVE (20 MARKS)

a) Let A be a 4×4 matrix with $|Adj(A)| = -8$. Find 2Aadj(2A). (5marks)

a) Let A be a non-singular 4×4 matrix with $|A^{-1}| = 3$. Find

i. |adj(A)| (4marks)

ii. $|\frac{1}{2}A^T Adj(A^{-1})|$ (6marks)

b) Find all vectors in $\mathbb{R}^4$ which are perpendicular to the vectors $X = (1, 1, 2, 2)$ and $Y = (2, 3, 5, 5)$. (5marks)