(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

COURSE CODE: **MAT 832**

COURSE TITLE: **RINGS AND MODULES**

DATE: 12/10/21 **TIME:** 9 AM -12 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

QUESTION ONE (20 MARKS)

a. Define the following
   i. Commutative ring (1mark)
   ii. Division ring (1 mark)
   iii. Field (1 mark)

b. Show that if R is a ring then
   i. The zero element is unique (1 mark)
   ii. The negative of any element is unique (1mark)
   iii. The unit is unique (1mark)

c. Let R be a ring. Show that
   i. 0a = a0 = 0 (3marks)
   ii. (-a)b = a(-b) = -ab (3 marks)

d. Show that all ideals of $\mathbb{Z}$ are of the form $\mathbb{Z}_n$ for some $\in \mathbb{Z}$. (8 marks)

QUESTION TWO (20 MARKS)

a. Define the following
   i. An ideal generated by F (1mark)
   ii. An ideal (2 mark)
   iii. Subring (2marks)
   iv. Ring homomorphism (3marks)

b. Let R be a ring and let $(I_t)_{t \in T}$ be a collection of ideals in R. Show that $\cap_{t \in T} I_t$ is an ideal in R. (7marks)

c. Let p be a prime number and a and b be integers. Show that if p\ab then p\a and p\b (5marks)

QUESTION THREE (20 MARKS)

a. Define the following
   i. Kernel of φ (1mark)
   ii. Image of φ (1mark)

b. Let $\varphi: R \rightarrow S$ be a ring homomorphism. Show that
   i. $\ker\varphi \subset R$ is an ideal (4marks)
   ii. $\operatorname{im}\varphi \subset S$ is a subring (4marks)

c. Show that a ring homomorphism $\varphi: R \rightarrow S$ is injective if and only if $\ker\varphi = 0$ (5marks)

d. Let R be a principal ideal domain and $a, b \in R$. Show that d = gcd (a, b) if and only if (a, b) = (d). (5marks)

QUESTION FOUR (20 MARKS)

a. Define the following
    i. Integral domain (1mark)
    ii. Zero divisor (1mark)
    iii. Invertible (1mark)
    iv. Associates (1mark)
    v. Principal ideal (1mark)
    vi. Least common multiple (3 marks)

b. Let R be a principal ideal domain. Show that any irreducible element in R is prime(6 marks)

c. Let R be an integral domain. Show that two elements $a, b \in R$ are associates if and only if (a) = (b). (6marks)

QUESTION FIVE (20 MARKS)

a. Define the following
    i. Submodule (2marks)
    ii. Indecomposable module (2marks)

b. Let K be a field and $p \in K[x]$ be irreducible. Show that $K[x]/(p)$ is a field (6 marks)

c. Let L be a field. Show that an intersection of a collection of subfields of L is a field (6marks)

d. Show that for any simple R-module M, the endomorphism ring is a division ring (4marks)