(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUATION AND BACHELOR OF SCIENCE

COURSE CODE: MAA 111

COURSE TITLE: DIFFERENTIAL CALCULUS

DATE: 24/09/2021 **TIME:** 11:00 AM- 1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

1. Show that limit of the following function does not exist

$$\text{Lim}_{x\to 0}\frac{|x|}{x}$$ (3mks)

a) Evaluate the following limits

i. $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x^3+8}$ (4mks)

ii. $\lim_{x\to\infty}\frac{(2x-3)(3x+5)(4x-6)}{3x^3+x-1}$ (3mks)

b) Evaluate

$$\lim_{x\to 0}\frac{\tan\alpha x}{\sin\beta x}$$ (3mks)

c) Prove that: $\lim_{x\to 3} 4x-5=7$ (3mks)

d) Determine if the following function is continuous at $x=4$

$$f(x)=\begin{cases} x^2-6 & x<4 \\ 10 & x=4 \\ x+6 & x>4 \end{cases}$$ (4 mks)

e) Find from the 1st principles or using the delta method the derivative of $y=\sqrt{x}$ (4mks)

f) Find the equation of the tangent and normal to the curve $y=\frac{4}{x}$ at the point (3,11)(5mks)

g) Find $\frac{d^4y}{dt^4}$ given that $y=6t^3+8t^{\frac{1}{2}}+e^{2t}$ (3mks)

QUESTION TWO (20 MARKS)

a) Prove the following

i. $\lim_{\alpha\to 0}\frac{sin\alpha}{\alpha}=0$ (5mks)

ii. $\lim_{x\to 2}\frac{x^2-4}{x-2}=4$ (5mks)

b) Evaluate the following limits

$$\lim_{x\to 0}\frac{1-cos2x}{xsinx}$$ (5mks)

QUESTION THREE (20 MARKS)

a) Find from the 1st principles or using the delta method the derivative of

$$y=6x^3-9x^2+2x+4$$ (5mks)

b) Find the slope of the line tangent to the graph of the equation $x^3+y^3-2xy^2+yx+2y=1+y^2$ at the point $(\frac{1}{2},\frac{1}{3})$ (5mks)

c) Find $\frac{dy}{dx}$ if $y = (2x+1)^7(x^2 - x + 1)^4$ (5mks)

d) Find the Cartesian equation for each of the following parametric form.

$x = \frac{1}{1+t}; y = t^2 + 4$ (5mks)

QUESTION FOUR (20 MARKS)

a) Show that the slope of the line tangent to the graph of the equation

$sinxy = x^2 cosy$ at $(2, \frac{\pi}{2})$ is $\frac{\pi}{4}$ (10mks)

b) If $y = \frac{sinx}{x^2}$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ hence prove that

$x^4\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} + (x^3 + 2x)y = 0$ (10mks)

QUESTION FIVE (20 MARKS

a) An object is moving vertically according to the equation $S = 250t - t^3$ where $t$the time in seconds is and S is the height of the object above the ground in feet.

(i) Find the velocity of the object when $t = 4$ seconds

(ii) What is the time when the object starts to move downwards?

(iii) How high does the object go (6mks)

b) Given the equation of the curve $y = \frac{x^4}{4} - \frac{4}{3}x^3 + \frac{x^2}{2} + 6x + 4$

Investigate the nature of the stationary points hence plot the curve. (10mks)

c) A cuboid of volume 30 M^3 has a square base x M. It is enclosed at the top and bottom. Determine the dimensions of the cuboid that will give maximum S.A (4mks)