(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 422**

COURSE TITLE: PDE II

DATE: 6/10/2021 **TIME:** 2:00 PM - 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE COMPULSORY (30 MARKS)

(a) Define the following terms giving an example in each;

i. Linear homogeneous partial differential equation. (4 marks)

ii. Linear non-homogeneous partial differential equation.

(b) Solve the wave equation by D'Alembert's method (6 marks)

$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$ where C is a constant

(c) Using method of characteristics solve the semi-linear partial differential equation. (8 marks)

$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = u$ given $u = x^2$ on $y = x, 1 \leq y \leq 2$

(d) Determine the complete solution of

i. $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = e^{2x+3y}$ (6 marks)

ii. $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x^2 + y$ (6 marks)

QUESTION TWO (20MARKS)

(a) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ under the conditions $u = 0$, when $x = 0$ and $x = \pi, \frac{\partial u}{\partial t} = 0$ when $t = 0$ and $u(x, 0) = x, 0 < x < \pi$ (13 marks)

(b) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} - 2z = 0$ (7 marks)

QUESTION THREE (20 MARKS)

(a) Distinguish between reducible and irreducible linear differential operator. (2 marks)

(b) Reduce the partial differential equation to its canonical form (10 marks)

$$y^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{xy}\left(y^3\frac{\partial u}{\partial x} + x^3\frac{\partial u}{\partial y}\right)$$

(c) Solve $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$ (8 marks)

QUESTION FOUR (20 MARKS)

(a) A rod of length L with insulated sides is initially at uniform temperature $100^0 C$. Its ends are suddenly cooled to $0^0 C$ and are kept at that temperature, find the temperature function $u(x, t)$. (13 marks)

(b) Determine the characteristic curves for $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial y^2} + x + y = 0$ (4 marks)

(c) Solve $\frac{\partial z}{\partial x} - \frac{\partial^2 z}{\partial y^2} = 0$ (3 marks)

QUESTION FIVE (20MARKS)

Given the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} + 1 = 0 \text{ in } 0 \le x \le 1, y > 0 \text{ with } u = \frac{\partial u}{\partial x} = x \text{ on } y = 0$$

i. Classification the PDE (2 marks)

ii. Reduce it to canonical forms and solve it (18 marks)