



KIBABII UNIVERSITY

**UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR**

SECOND YEAR SECOND SEMESTER MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: STA 221/STA 242

COURSE TITLE: PROBABILITY AND DISTRIBUTION MODELS

DATE: 6/10/2021

TIME: 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One in and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS) COMPULSORY

(a) Define the term moment generating function and hence or otherwise show step by step how the mean and variance of a density function $f(x,y)$ can be obtained. (4mks)

(b) Find the value of r and hence the quartile deviation of a pdf defined as (6mks)

$$f(z) = r(3 - z^2) \text{ where } 0 < z < 3$$

(c) A catalyst researcher states that the diameter, in microns, of the pores in a new product she has made has exponential distribution with parameter $\eta = 2$. Determine the mean and variance of the pore diameter? (6mks)

(d) For a chi-square distribution with r degree of freedom and its distribution density given as;

$$f(x) = \begin{cases} \frac{1}{2^{r/2} \Gamma(r/2)} \cdot x^{\frac{r-2}{2}} \cdot e^{-\frac{x}{2}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Show that moment generating function,

$$M_x(t) = (1 - 2t)^{-r/2} \quad (7mks)$$

Hence or otherwise using the moment generating function, find (3mks)

- i) $E(x)$ (4mks)
- ii) $\text{Var}(x)$.

QUESTION TWO (20 MARKS)

(a). Two distinct random variables x and y have joint density functions;

$$f(x,y) = \begin{cases} \lambda^x e^{-\lambda} p^y (1-p)^{x-y}, & y = 0, 1, 2, \dots, x \\ & x = 0, 1, 2, \dots, \infty \\ & x \geq y \\ 0, & \text{elsewhere} \end{cases}$$

where λ, p are constants with $\lambda > 0, 0 < p < 1$

Find; (5mks)

- (i). the marginal pdf of x and y . (3mks)
- (ii). conditional density of Y given $X=x$.

(b) The joint distribution function is given as:

$$f(x,y) = \frac{3}{7} (x - y); \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 2 \quad (8mks)$$

- i) Find the correlation coefficient between the random variables (4mks)
- ii) Determine the value of $\text{Var}(x + 2y - 3)$

QUESTION THREE (20MARKS)

a) Given the joint density of X_1 and X_2 as

$$f(x, y) = \begin{cases} 1 & 0 < x_1 < 1 \quad 0 < x_2 < 1 \\ 0 & \text{Otherwise} \end{cases}$$

Find

- i) The joint density of $Y = X_1 + X_2$ and $Z = X_2$ (4mks)
- ii) The marginal density of Y (3mks)

(b) For a gamma distribution with parameters α and β ;

$$f(y, \alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} y^{\alpha-1} e^{-\frac{y}{\beta}} \quad ; y \geq 0, \beta \geq 0$$

Show that moment generating function,

$$M_x(t) = \frac{1}{(1 - \beta t)^\alpha} \quad \text{if}$$

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy \quad \text{with } y = \frac{x}{\beta} \quad \text{where } \beta > 0$$

(8mks)

Hence or otherwise using the moment generating function, find the mean and variance of a gamma distribution.

(5mks)

QUESTION FOUR (20 MARKS)

a) Given that $X \sim \text{Exp}(\beta)$ ie $f(x, \beta) = \frac{1}{\beta} e^{-x/\beta}$, $x > 0$. Find the moment generating function of the distribution and hence its mean and variance. (10mks)

b) Consider the following bivariate function defined by;

$$f(x, y) = \begin{cases} k(6 - x - y) & 0 < x < 2, 2 < y < 4 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the value of the constant k such that $f(x, y)$ is the pdf of the two random variables X and Y . Hence evaluate the following.

(i). $p(x \leq 1, y \leq 3)$

(ii). $p(x + y < 3)$

(10mks)

QUESTION FIVE (20 MARKS)

Earthquakes are natural disasters that cause millions of dollars in damages each year. An actuary scientist determines that the annual number of earthquakes in two counties: Kisumu and Busia are jointly distributed as indicated in the table below.

Let Y_1 and Y_2 denote the number of earthquakes seen each year in Kisumu and Busia counties respectively

$P(y_1, y_2)$	$Y_2 = 0$	$Y_2 = 1$	$Y_2 = 2$	$Y_2 = 3$
$Y_1 = 0$	0.12	0.06	0.05	0.02
$Y_1 = 1$	0.13	0.15	0.12	0.03
$Y_1 = 2$	0.05	0.15	0.10	0.02

- (a) What is the probability that there is no more than one earthquake in the two counties combined (4mks)
- (b) What is the probability that there are two earthquakes in Busia county (4mks)
- (c) Obtain the mean number of earthquakes the two counties 4mks)
- (d) Determine if the occurrence of earthquakes in the two counties is correlated (8mks)