(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREES OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: MAP221

COURSE TITLE: LINEAR ALGEBRA II

DATE: 6TH OCTOBER, 2021 **TIME: 9:00AM-11:00AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO (2) Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

SECTION A (COMPULSORY)

Question One (30 Marks)

a) Let s(x,y,z) = (x+2y-3z, 2x+y+z, 5x-y+z). Find the co-ordinates of an arbitrary vector (a,b,c) ϵ IR^3 with respect to the basis B = { u_1 = (1,1,0), u_2 = (1,2,3), u_3 = (1,3,5)} (6 marks)

a) State any three properties of a Euclidean space (3 marks)

b) Show that the usual basis of Euclidean space IR^3:E={ e_1 = (0,1,0), e_2 = (1,0,0)and e_3 =(0,0,1)} form an orthornormal set in IR^3 with the Euclidean inner product. (9 marks)

c) Let F:IR^3 → IR^3 be defined by F(x,y,z) = (2x-3y+4z, 5x-y+2z, 4x+7y). Find the matrix of F relative to the standard basis of IR^3 E = { e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1) } (10 marks)

d) Find the eigen values of the following characteristic of a matrix (2 marks)

$$\begin{vmatrix} \lambda + 2 & 1 \\ -5 & \lambda - 2 \end{vmatrix} = 0$$

SECTION B (ANSWER ANY TWO QUESTIONS)

Question Two (20 Marks)

a) Find an orthonormal basis for the subspaces U of IR^4 spanned by v_2 = (1,2,4,5) v_1 = (1,1,1,1) and v_3 = (1,-3,-4,-2) (10 marks)

b) Normalize the orthogonal set S = { u = (1,2,-3,4) v = (3,4,1,-2) and w = (3, -2, 1,1) } to obtain an orthonormal set. (10 marks)

Question Three (20 marks)

Let S_1 = { u_1 = (1,-2), e_2 = (3,-4) }

S_2 = { v_1 = (1,3), v_2 = (3,8) }

A) Find the change of basis matrix P from S_1 to S_2

B) Find the change of basis matrix Q from S_2 to S_1

C) Verify that $Q=P^{-1}$

D) Show that $P[\vec{V}]S_2=[V]S_1$

Question Four (20 Marks)

Let $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

i) Find the characteristic equation of A

ii) Find the bases for the eigen spaces of the matrix A (20 marks)

Question Five (20 Marks)

a) Find the quadratic form of A given that

$A = \begin{bmatrix} 5 & -5 \\ -5 & 1 \end{bmatrix}$ (5 marks)

b) Show that $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is a positive matrix (7 marks)

c) Find the co-ordinates of an arbitrary vector (a, b,c) in IR^3 with respect to the basis

$s_1 = \{ u_1 = (1,2,0), u_2 = (1,3,2) \text{ and } u_3=(0,1,3)\}$

$S_2 = \{v_1 = (1,2,1), v_2 = (0,1,2) \text{ and } v_3=(1,4,6)\}$

Hence find the change of basis matrix P from S_1 to S_2 (8 marks)