(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

COURSE CODE: MAT 830

COURSE TITLE: REPRESENTATION THEORY OF GROUPS

DATE: 8/10/21 **TIME:** 9 AM -12 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

MAT 843: REPRESENTATION THEORY EXAM

August 2021

Answer any Three questions neately and precisely. Each question carries 20 marks

Question One

(a) Define the term representation of a finite group G (2mks)
(b) Let G be a group of permutations of $X = \{1, 2, 3, \cdots n\}$. For an ordered basis $\{v_1, v_2, \cdots v_n\}$ basis of an $n-$dimensional vector space V over any field F, define $T(g)v_i = v_{g(i)}$. Show that T is a representationof G on V. (4mks)
(c) Let S_3 be the Symmetric group on 3 points $\{1, 2, 3\}$ generated by two elements (12) and (123).
(i) Find the permutation representation of the generators of S_3. (6 mks)
(ii) Write out permutation matrix representation of S_3 of degree 2. (It is enough to give the matrix representations of the generators.) (8 mks)

Question Two

(a) Let G be a finite group. State the row and column orthogonality relations for a character table. (4 mks)
(b) For the rest of the question, G will be a group of order 20 with 5 conjugacy classes. Here are the first two lines of its character table.

$g =$

	$g_1 = e$	g_2	g_3	g_4	g_5
$\lvert C_i \rvert$	1	4	5		5
χ_1	1	1	1	1	1
χ_2	1	1	i	-1	i

(i) What are the dimensions of the remaining representation of G. (4 mks)
(ii) Find aanother $1-$dimensional representation of G (4mks)
(iii) Find the remaining entries of the character table. (8 mks)

Question Three

(a)If T is a representation of G Define on V. Define what is meant by the following;
(i) A $T-$invariant subspace. (2mks)

(ii) T is irreducible (2mks)
(iii) T is indecomposable (2mks)
(iv) T is completely reducible (2 mks)
(b) Suppose $F \subseteq \mathbb{C}$ and $\hat{T}$ is a $F-$representation of a group G. Define $\hat{T}^*$, the contragradient representation of T via

$$\hat{T}^* = (\hat{T}(x^{-1}))^t$$

for every $x \in G$, where t represents the transpose of a matrix.
Show that $\hat{T}$ is a matrix representation of G [i.e. $\hat{T}^*(xy) = \hat{T}^*(x)\hat{T}^*(y)\ \forall x, y \in G$] (4 mks)
(c) Compute the character table of $G = C_2 \times C_2$ (8mks)

Question Four

(a) Define what is meant by the following
(i) A character of a representation T of a group G. (2 mks)
(ii) An irreducible character χ of a representation. (2mks)
(b) Let χ_1 and χ_2 be two $F-$ characters of G. Give the equation of the inner product of the two characters $< \chi_1, \chi_2 >$ (3 mks)
(c) Show that characters are class functions i.e $\chi(y^{-1}xy) = \chi(x)$ (4mks)
(d)(i) Compute the character table of the Quartenion group $G = Q_8 = \{\pm 1, \pm i \pm j, \pm k\}$ of order 8. (8 mks) (ii) State the order of the commutator subgroup of Q_8 (1mks)

Question Five

(a) Let S and T be two representations of G.
(i) what is meant by saying S is equivalent to T (2 mks)
(ii) If S and T are equivalent $F-$representations of G with characters χ and ψ respectively. Show that $\chi = \psi$ (4mks)
(b)Let G be the Alternating group A_4.
(i) Write down the conjugacy classes of A_4 (4mks)
(ii) compute the character table of $G = A_4$ (10mks)