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UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF BSC (PHYSICS)

COURSE CODE: SPH 414

COURSE TITLE: QUANTUM MECHANICS II

DURATION: 2 HOURS

DATE: 5/10/2021 **TIME: 8:00-10:00AM**

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other two (2) Questions.
- Indicate **answered questions** on the front cover.

Start every question on a new page and make sure question's number is written on each page
This paper consists of 3 printed pages. Please Turn Over

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The following rules of commutator algebra may be used where necessary

$$[A,B] = AB - BA = -[B,A] = 0$$

$$[A, B + C] = [A,B] + [A,C]$$

$$[A + B, C] = [A,C] + [B,C]$$

$$[A, BC] = [A,B]C + B[A,C]$$

$$[AB, C] = A[B,C] + [A,C]B$$

$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$$

QUESTION ONE [30 Marks]

a) Show that any two components of the spin angular momentum operator are not simultaneously measurable. [6 Marks]

b) Simplify the arbitrary products of spin half operators; S_xS_y, S_zS_y, S_zS_x. [3 Marks]

c) Show that;

i) $J_+J_- = J^2 - J_z^2 + \hbar J_z$ [3 Marks]

$S_xS_y = \frac{1}{2} i\hbar S_z$ [2 Marks]

ii) Find the commutators; $[\sigma_+\sigma_-]$ [2 Marks]

d) Verify that $S^2\alpha = \frac{3}{4}\hbar^2\alpha$ and $S_z\alpha = \frac{\hbar}{2}\beta$, where symbols have their usual meaning. [4 Marks]

e) Show that;

i) $[J_z, J_+] = J_+$ (iii) $[J_x, J_y] = iJ_z$ [4 Marks]

f) State the Pauli spin matrices σ_x σ_y σ_z and show that $\sigma_x \sigma_y = -\sigma_y \sigma_x = i\sigma_z$ [6 Marks]

QUESTION TWO [20 Marks]

a) Write down an expression for the z-component of angular momentum, L_z, of a particle moving in the (x, y) plane in terms of its linear momentum components p_x and p_y. [2Marks]

b) Using the operator correspondence $P_x = -i\hbar\frac{\partial}{\partial x}$ etc., show that;

$$L_Z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

Hence show that $L_Z = -i\hbar\frac{\partial}{\partial\varphi}$, where the coordinates (x,y) and (r, φ)are related in the usual way. [4Marks]

c) Given that $L = r \times p$, show that $\{L_x, L_y\} = i\hbar L_z$ [7Marks]

d) Use the rules of the commuter algebra to find the values of; $\{L_x, p_y\}$ and $\{L^2, L_x\}$ [7Marks]

QUESTION THREE [20 Marks]

Obtain the angular momentum matrices for J^2, J_z, J_x and J_y for the case of j =1 particles. [20 Marks]

QUESTION FOUR [20 Marks]

a) What is perturbation theory? [2 Marks]

b) Show that the first order correction energy $E^{(1)}$ for a non degenerate level is just a perturbation H' averaged over the corresponding unperturbed state of the system. [12 Marks]

c) Given the unperturbed Hamiltonian for the linear harmonic oscillator $H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2, k > 0$ and the unperturbed energy levels $E_n^{(0)} = \hbar\omega\left(n + \frac{1}{2}\right)$ n=0, 1, 2 …, Write the perturbed eigen functions and eigen values [6 Marks]

QUESTION FIVE [20 Marks]

a) Write down the Pauli matrices σ_x, σ_y and σ_z [3Marks]

b) Obtain the values of σ_x^2, σ_y^3 and $\sigma_x^2 + \sigma_y^2 + \sigma_z^2$ [6Marks]

c) Obtain the values of $\sigma_x\sigma_y$, $\sigma_y\sigma_z$ and $\sigma_z\sigma_x$ [6Marks]

d) Show that [5Marks]

$$J^2 = \frac{3}{4}\hbar^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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