(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 402**

COURSE TITLE: **TOPOLOGY II**

DATE: 6/10/2021 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following terms
   (i). Connected space (2 mks)
   (ii). Second countable space (2 mks)
   (iii). Hausdorff space (2 mks)
   (iv). T_1 space (2 mks)
   (v). Normal space (2 mks)

b) Let $X = [0,2]$, show that we cannot have a separation of X. (3 mks)

c) Show that the image of a compact set under a continuous map is compact. (4 mks)

d) Show that a subspace of a second countable space is second countable. (4 mks)

e) (i). Let $X = \{a, b\}$. Form a topological space, τ_x,(non trivial) on X. (1 mk)
   (ii). Show that τ_x is a topology but not a T_1 topological space. (4 mks)

f) Let X be a regular space and one-point sets be closed in X. Show that if given a point $x \in X$ and a neighbourhood U of x, there is a neighbourhood V of x such that $\bar{V} \subset U$. (4 mks)

QUESTION TWO (20 MARKS)

a) Define the term a compact space, hence show that space of real numbers, $\mathbb{R}$, is not compact. (4 mks)

b) Show that the set $X = \{x, y, z, k\}$ with a topology $\tau = \{\phi, \{z\}, \{y, z\}, X\}$ is not a Hausdorff space. (3 mks)

c) Let X and Y be topological space and $f: X \to Y$ a continuous function. If X is connected, show that $f(X)$ is connected. (5 mks)

d) State and prove the intermediate value theorem (8 mks)

QUESTION THREE (20 MARKS)

a) (i).What is a linear continuum? (2 mks)
   (ii).Prove that a linear continuum L, in the order topology, as well as its intervals and rays are connected. (11 mks)

b) Prove that every metrizable space is normal. (7 mks)

QUESTION FOUR (20 MARKS)

a) Show that the real line $\mathbb{R}$ is locally compact. (4 mks)

b) Prove the claim that the interval $[0,1]$ on the real line is sequentially compact. (5 mks)

c) Let $A = X \times [0,1]$ be a product topological space . Show that A is a linear continuum. (6 mks)

d) Prove that the real line, $\mathbb{R}$, together with the usual matrix $d = |x - y|$ for $x, y \in \mathbb{R}$ is a Hausdorff space. (5 mks)

QUESTION FIVE (20 MARKS)

a) Let X be a topological space. Define a relation $x \sim y$ on X if there is a connected subspace of X containing both x and y. Show that $\sim$ is an equivalence relation. (5 mks)

b) Prove that a topological space X is a T_1 space if an only if every singleton subset of X is closed. (5 mks)

c) Let $\{A_\alpha\}, \alpha \in I$ be a collection of connected subspaces with a common point. Show that $\bigcup_{\alpha \in I} A_\alpha$is connected. (5 mks)

d) When is a function said to be uniformly continuous on a metric space (X, d) where d is a metric on X. Provide an example of uniformly continuous function and one that is not is continuous but not uniformly continuous on a given interval. (5 mks)