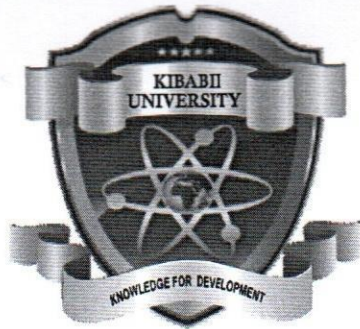


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*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2020/2021 ACADEMIC YEAR**  
**FOURTH YEAR SECOND SEMESTER**  
**MAIN EXAMINATION**  
**FOR THE DEGREE OF BACHELOR OF SCIENCE**  
**(MATHEMATICS)**

**COURSE CODE: MAT 402**

**COURSE TITLE: TOPOLOGY II**

**DATE: 6/10/2021**

**TIME: 9:00 AM - 11:00 AM**

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

### QUESTION ONE (30 MARKS)

- a) Define the following terms (2 mks)
- (i). Connected space (2 mks)
  - (ii). Second countable space (2 mks)
  - (iii). Hausdorff space (2 mks)
  - (iv).  $T_1$  space (2 mks)
  - (v). Normal space (3 mks)
- b) Let  $X = [0,2]$ , show that we cannot have a separation of  $X$ . (4 mks)
- c) Show that the image of a compact set under a continuous map is compact. (4 mks)
- d) Show that a subspace of a second countable space is second countable. (4 mks)
- e) (i). Let  $X = \{a, b\}$ . Form a topological space,  $\tau_x$ , (non trivial) on  $X$ . (1 mk)
- (ii). Show that  $\tau_x$  is a topology but not a  $T_1$  topological space. (4 mks)
- f) Let  $X$  be a regular space and one-point sets be closed in  $X$ . Show that if given a point  $x \in X$  and a neighbourhood  $U$  of  $x$ , there is a neighbourhood  $V$  of  $x$  such that  $\bar{V} \subset U$ . (4 mks)

### QUESTION TWO (20 MARKS)

- a) Define the term a compact space, hence show that space of real numbers,  $\mathbb{R}$ , is not compact. (4 mks)
- b) Show that the set  $X = \{x, y, z, k\}$  with a topology  $\tau = \{\emptyset, \{z\}, \{y, z\}, X\}$  is not a Hausdorff space. (3 mks)
- c) Let  $X$  and  $Y$  be topological space and  $f: X \rightarrow Y$  a continuous function. If  $X$  is connected, show that  $f(X)$  is connected. (5 mks)
- d) State and prove the intermediate value theorem (8 mks)

### QUESTION THREE (20 MARKS)

- a) (i). What is a linear continuum? (2 mks)
- (ii). Prove that a linear continuum  $L$ , in the order topology, as well as its intervals and rays are connected. (11 mks)
- b) Prove that every metrizable space is normal. (7 mks)

#### QUESTION FOUR (20 MARKS)

- a) Show that the real line  $\mathbb{R}$  is locally compact. (4 mks)
- b) Prove the claim that the interval  $[0,1]$  on the real line is sequentially compact. (5 mks)
- c) Let  $A = X \times [0,1]$  be a product topological space. Show that  $A$  is a linear continuum. (6 mks)
- d) Prove that the real line,  $\mathbb{R}$ , together with the usual metric  $d = |x - y|$  for  $x, y \in \mathbb{R}$  is a Hausdorff space. (5 mks)

#### QUESTION FIVE (20 MARKS)

- a) Let  $X$  be a topological space. Define a relation  $x \sim y$  on  $X$  if there is a connected subspace of  $X$  containing both  $x$  and  $y$ . Show that  $\sim$  is an equivalence relation. (5 mks)
- b) Prove that a topological space  $X$  is a  $T_1$  space if and only if every singleton subset of  $X$  is closed. (5 mks)
- c) Let  $\{A_\alpha\}, \alpha \in I$  be a collection of connected subspaces with a common point. Show that  $\bigcup_{\alpha \in I} A_\alpha$  is connected. (5 mks)
- d) When is a function said to be uniformly continuous on a metric space  $(X, d)$  where  $d$  is a metric on  $X$ . Provide an example of uniformly continuous function and one that is not uniformly continuous but continuous on a given interval. (5 mks)