(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 402**

COURSE TITLE: TOPOLOGY II

DATE: 02/02/2021 **TIME:** **11 AM -1 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms

I. Normal space (2 mks)

II. Compact space X. (2 mks)

III. Unit sphere in $\mathbb{R}^n$ (2 mks)

IV. 2^{nd} countable space (2 mks)

b) Prove that every subspace of a second countable space is second countable(6mks)

c) Let $f: X \to Y$ be a continuous function from a topological spaces, X, to another, Y. Show that its image is compact if X is compact. (6 mks)

d) (i).Show that the subspace $Y = [0,1]$ of a real line is connected. (5 mks)

(ii). Give an example of a subspace of Y that is not connected (1 mk)

e) What do you understand by the term a complete regular space? Give an example (4mks)

QUESTION TWO (20 MKS)

a) Define a T_2 space given an example. (3 mks)

b) What is path connected space? Give an example (3 mks)

c) Define the term a compact space, hence show that space of real numbers, $\mathbb{R}$, is not compact. (5 mks)

d) Every metrizable space is normal. (9 mks)

QUESTION THREE (20 MARKS)

a) Show that the interval $B = (0,1)$ of the real line with the usual topology is not sequentially compact. (5 mks)

b) Let $\{A_i\}i \in I$ be a collection of connected subspaces with a common point. Show that $\bigcup_{i\in I} A_i$is connected. (5 mks)

c) Prove that every open covering of a space X with a countable basis contains a countable sub collection covering X. (6 mks)

d) Find the smallest compact set containing (p, q) given that $p, q \in \mathbb{R}$. Can there be a separation of the new set determined? Explain? (4 mks)

QUESTION FOUR (20 MARKS)

a) What is a linear continuum? (2 mks)

b) Let $I \times I$ be a product topological space and π_1, π_2 be projections on I respectively be defined as $\pi_1(x, y) = x$ and $\pi_2(x, y) = y$ for $x, y \in I$. Let $A \subset I \times I$ be square $A = \{x, y: a \leq x \leq b, c \leq y \leq d, a, b, c, d \in \mathbb{R}\}$. Show that A is a linear continuum. (8 mks)

c) State and prove the generalization of extreme value theorem. (10 mks)

QUESTION FIVE (20 MARKS)

a) Show that the space $\mathbb{R}_l$ is normal. (5 mks)

b) Let X be a topological space. Define a relation $x \sim y$ on X if there is a connected subspace of X containing both x and y. Show that $\sim$ is an equivalence relation. (6 mks)

c) When is a collection of subsets of a space X said to have a finite intersection property? (2 mks)

d) Define a set X as $X = \{a, b\}$. For a T_1 topological space from the set. Show that space formed is a topological spaces but notT_1. (7 mks)