(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 404

COURSE TITLE: DIFFERENTIAL TOPOLOGY

DATE: 09/02/2021   **TIME:** 11 AM – 1 PM

INSTRUCTIONS TO CANDIDATES
Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION 1 (30 MARKS)

a) Define a manifold (2 marks)

b) Show that $X \times Y \subset \mathbb{R}^M \times \mathbb{R}^N$ is a smooth manifold (8 marks)

c) Let $Z = f^{-1}(y)$ for a regular value y of the mapping $f: X \to Y$. Prove that
$$Ker\,[\,df_x: T_x X \to T_y Y\,] = T_x Z \text{ at any point } x \in Z.$$ (8 marks)

d) Prove that a subspace $M \subset \mathbb{R}^k$ is a smooth manifold of dimension n if every point in M has an open neighborhood in M which is diffeomorphic to an open subset of $\mathbb{R}^n$. (6 marks)

e) Show that if $\dim(M) = \dim(N)$ and $y \in N$ is a regular value of f then $f^{-1}(y)$ is a discrete set. Furthermore if M is compact then $f^{-1}(y)$ is a finite set. (6 marks)

QUESTION 2 (20 MARKS)

a) Show that the circle $S^1 = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ is a one-dimensional manifold. (8 marks)

b) Prove that if M is an $m-$dimensional manifold then the tangent space $T_x(M)$ is an $m-$dimensional linear space. (6 marks)

c) State the Inverse function theorem without prove. (2 marks)

d) Suppose that $f: X \to Y$ is a diffeomorphism. Prove that $df_x: T_x(X) \to T_{f(x)}(Y)$ is a linear isomorphism. (4 marks)

QUESTION 3 (20 MARKS)

a) Prove that an embedding $f: X \to Y$ maps X diffeomorphically into a submanifold of Y. (10 marks)

b) Show that every $k-$dimensional manifolds admits a one-to-one immersion in $\mathbb{R}^{2k+1}$ (6 marks)

c) Show that on any manifold X there exists a proper map $\rho: X \to \mathbb{R}$ (4 marks)