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KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER SUPPLEMENTARY EXAMINATIONS

FOR THE DEGREE OF BSC (PHYSICS)

COURSE CODE: SPH 414

COURSE TITLE: QUANTUM MECHANICS II

DURATION: 2 HOURS

DATE: ~~APRIL 2020~~ 1/2/2021 **TIME:** 2-4pm

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other two (2) Questions.
- Indicate **answered questions** on the front cover.

Start every question on a new page and make sure question's number is written on each page
This paper consists of **3** printed pages. Please Turn Over

KIBU observes ZERO tolerance to examination cheating

The following rules of commutator algebra may be used where necessary

$$[A, B] = AB - BA = -[B, A] = 0$$

$$[A, B + C] = [A, B] + [A, C]$$

$$[A + B, C] = [A, C] + [B, C]$$

$$[A, BC] = [A, B]C + B[A, C]$$

$$[AB, C] = A[B, C] + [A, C]B$$

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$$

QUESTION ONE [30 Marks]

a) Using a test function f(x), show that the canonical commutation relation between distance x and linear momentum p is given by $[x,p] = i\hbar$ (4mks)

b) Angular momentum is represented classically by $\overline{L} = \overline{r} \times \overline{p}$ show that orbital angular momentum is represented in the position representation of wave mechanics by the vector operator $\overline{L} = -i\hbar(\overline{r} \times \nabla)$. (5mks)

c) Show that angular momentum L, is self-adjoint i.e. $L_x = L_x^\dagger, L_y = L_y^\dagger, L_z = L_z^\dagger$. (3mks)

d) Define the term perturbation? (2mks)

e) List two categories of perturbation theory? (2mks)

f) Reduce the following arbitrary product of spin ½ operators?

i) $S_xS_yS_zS_yS_zS_x$ (4mks)

ii) $S_xS_yS_xS_yS_zS_x$ (4mks)

g) Show that

i. $s^2\alpha = \frac{3}{4}\hbar^2\alpha$ (2mks)

ii. $s_z\alpha = \frac{1}{2}\hbar\alpha$ (2mks)

QUESTION TWO [20 Marks]

Using canonical commutation relation and the commutator algebraic relation to show that

a) $\left[L_x, L_y\right] = i\hbar L_z$ (7mks)

b) $\left[L_x, y\right] = i\hbar z$ (6mks)

(a) $\left[L_x, p_y\right] = i\hbar p_z$ (7mks)

QUESTION THREE [20 Marks]

Show that

a) $J^2 = \frac{3}{4}\hbar^2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (10mks)

b) $J_z = \frac{1}{2}\hbar\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ (10mks)

QUESTION FOUR [20 Marks]

a) The matrices representing S_x, S_y, S_z, which acts on the spin wave function c for S=1/2, are $s = \frac{1}{2}\hbar\sigma$ with $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Show that $[\sigma_x, \sigma_y] = 2i\sigma_z$ $[\sigma_z, \sigma_x] = 2i\sigma_y$, and $[\sigma_y, \sigma_z] = 2i\sigma_x$ (12mks)

b) Given the unperturbed Hamiltonian for the linear harmonic oscillator $H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2, k > 0$ and the unperturbed energy levels $E_n^{(0)} = \hbar\omega\left(n + \frac{1}{2}\right)$ n=0, 1, 2 …, Write the perturbed eigenfunctions and eigenvalues (8mks)

QUESTION FIVE [20 Marks]

a) Write down an expression for the z-component of angular momentum, L_z, of a particle moving in the (x, y) plane in terms of its linear momentum components p_x and p_y. (2mks)

b) Using the operator correspondence $P_x = -i\hbar\frac{\partial}{\partial x}$ etc., show that;

$$L_Z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

Hence show that $L_Z = -i\hbar\frac{\partial}{\partial\varphi}$, where the coordinates (x,y) and (r, φ)are related in the usual way. (4mks)

c) Assuming that the wave function for this particle can be written in the form $\psi(r, \varphi)$ $=R(r)\Phi(\varphi)$ show that the z-component of angular momentum is quantized with eigen Value $\hbar$, where m is an integer. (14mks)

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