(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 434

COURSE TITLE: DIFFERENTIAL GEOMETRY

DATE: 02/02/2021   **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30MARKS)

a). Define the following terms

    i). Smooth curve   (1 mk)

    ii). Regular of a curve   (1 mk)

    iii). Helix   (1 mk)

    iv). Torsion to a curve   (1 mk)

b). State and derive the second fundamental form of a surface $X = X(u, v)$ whose class is more or equal to 2.   (6 mks)

c). Find the unit normal vector to the surface $X(u, \theta) = \langle u \cos \theta , u \sin \theta , 2\theta \rangle$. (6 mks)

d). Find the volume of the parallelepiped spanned by vectors $u = (3,3,7), v = (2,1,-1)$ and $w = (4,2,-3)$ respectively.   (4 mks)

e). Determine the first and the second curvature of the curve $r(t) = 2t\boldsymbol{i} + 4 \sin t\, \boldsymbol{j} + 4 \cos t\, \boldsymbol{k}$.

                                                  (10 mks)

QUESTION TWO (20MARKS)

a). Find the radius of curvature and the torsion to the circular helix $r(t) = \langle 4 \cos t , 4 \sin t , 2t \rangle$ at $t = \frac{\pi}{2}$.   (14 mks)

c). Let γ be a curve lying on the surface $X = X(u, v)$ where $u = u(t), v = v(t), a \leq t \leq b$. Prove that the length of the arc on the curve is given by $\int_a^b \sqrt{I}\ dt$ where I is the first fundamental form of a surface.   (6 mks)

QUESTION THREE (20MARKS)

a). Determine the lines of curvature to the helicoid $r(s, t) = \langle s \cos t , s \sin t , bt \rangle$.   (12 mks)

b). Determine the arc length of the curve $X(t) = \langle e^{2t} \cos t , e^{2t} \sin t , e^{2t} \rangle$ for $0 \leq t \leq \frac{\pi}{2}$.

                                                  (5 mks)

c). Given the equation of the surface $X(u, v) = \langle 2u, 2u, uv \rangle$ find its first fundamental form.

                                                  (3 mks)

QUESTION FOUR (20MARKS)

a). State and proof the Frenet – Serret formulas to the curve $X = X(s)$. (10 mks)

g). Find the equation of the tangent line and normal plane to the curve $X(t) = (1 + t)\hat{e}_1 - t^2\hat{e}_2 + (1 + t^3)\hat{e}_3$ at $t = 1$. (5 mks)

d). Find the equation of the rectifying plane $X(t) = \langle t^2, t^2, (1 + 4t) \rangle$ at $t = 1$. (5 mks)

QUESTION FIVE (20MARKS)

a). Find the unit binomial vector to the curve $X(t) = \langle 2t + 2t^3, 3t + \frac{t^2}{2}, 4t^2 \rangle$ at $t = 1$. (8 mks)

b). Let $X = X(u, v)$ be surface with directions given in parametric form as $(du : dv)$ and $(\delta u, \delta v)$ whose tangential vectors are $dX = X_u du + X_v dv$ and $\delta X = X_u \delta u + X_v \delta v$ respectively. Prove that the angle between the two directions is given by

$$\theta = \cos^{-1}\left(\frac{I(d, \delta)}{\sqrt{I(d)}\sqrt{I(\delta)}} \right)$$

(8 mks)

Where $I(d, \delta)$ is the first fundamental form of a surface.

c). Find the equation of the normal plane $X(t) = \langle t, -2t^2, (1 + t^2) \rangle$ at $t = 1$ (4 mks)