(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 426

COURSE TITLE: FOURIER SERIES

DATE: 8/10/2021 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE **(30 mks)**

(a) Compute the Fourier Series of f defined by $f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ 1 & 0 \le x \le \pi \end{cases}$ (6 mks)

(b) Draw the graph of $f(x) = x(10 - x)$ for $-10 < x < 10$.

(c) If $f(x) = x, 0 < x < 2$ the Fourier Series for $f(x)$ is given by
$x = \frac{4}{\pi}(\sin\frac{\pi x}{2} - \frac{1}{2}\sin\frac{2\pi x}{2} + \frac{1}{3}\sin\frac{3\pi x}{2}$ ....). (5 mks)
By integrating both sides determine the Fourier Series for $f(x) = x^2$ and show that $C = \frac{16}{\pi^2}(1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \ldots\ldots\ldots)$, where C is the constant of integration. (5 mks)

(d) Prove that $\int_{-k}^{k} \sin\frac{m\pi x}{k} \cos\frac{n\pi x}{k} dx = 0$ (4 mks)

(e) Sketch the functions $f(x) = \begin{cases} -\sin x & -\pi \le x \le 0 \\ \sin(-x) & 0 \le x \le \pi \end{cases}$ (4 mks)

(f) Show that an even function can have no sine terms in its Fourier Series expansion.

QUESTION TWO **(20 mks)**

(a) Prove that (i) $\int_{-k}^{k} \sin\left(\frac{m\pi x}{k}\right) dx = \int_{-k}^{k} \cos\left(\frac{m\pi x}{k}\right) dx = 0$ (3 mks)

(ii) $\int_{-k}^{k} \cos\frac{m\pi x}{k} \cos\frac{n\pi x}{k} dx = \int_{-k}^{k} \sin\frac{m\pi x}{k} \sin\frac{n\pi x}{k} dx = \begin{cases} 0 & m \ne n \\ k & m = n \end{cases}$ (14 mks)

(b) If the series $A + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right)$ converges uniformly to $f(x)$ in $(-l, l)$
Show that for $n = 1,2,3, \ldots\ldots$

(i) $a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right) dx$ (1 mk)

(ii) $b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right) dx$ (1 mk)

(iii) $a_0 = 2A$ (1 mk)

QUESTION THREE **(20 mks)**

(a) Using the Fourier Series for x^2, deduce $\frac{\pi^2}{6} = \sum_1^n \frac{1}{k^2}$ where $-\pi \le x \le \pi$ (15 mks)

(b) If $S_n(x) = 2\sum_1^{\infty} \frac{(-1)^n \sin nx}{n}$ draw the graph of $S_2(x)$ (5 mks)

QUESTION FOUR **(20 mks)**

(a) (i) Find the Complex Fourier Series for $F_C(x)$ if $F_R(x) = \sin^3 2x$ (4 mks)

(ii) Express the Real Fourier Series $F_R(x) = \cos^{-2} x$ in the Complex Fourier Series form $F_C(x)$ (8 mks)

(iii) If $f(x) = x^2$ determine $||f||$. Hence normalize the function f. (4 mks)

(b) If $D_n(\theta) = \frac{1}{2} + \cos\theta + cos2\theta + \cdots + \cos n\theta$, show that $\frac{\pi}{4}\int_0^{\pi} Dn\,(\theta)d\theta = \frac{\pi^2}{8}$ (4mks)

(20 mks)

QUESTION FIVE

a) Find the Fourier series expansion for $f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ sin^2x & 0 \leq x \leq \pi \end{cases}$ (14mks)

b) If f belongs to R_2 on $\{-\pi \leq x \leq \pi\}$ show that for each n, $\Delta_n = \int_{-\pi}^{\pi}[F(x) - S_n(x)]^2\,dx$ is a minimum if for $S_n(x) = \frac{C_o}{2} + \sum_{k=1}^{n}(C_k \cos kx + d_k \sin kx)$ then $c_k = a_k$ and $d_k = b_k$, where the $a_k{}'s\ and\ b_k{}'\,s$ are the Fourier coefficients of f. (6mks)