(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

THIRD YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 308

COURSE TITLE: RING THEORY

DATE: 09/02/2021 **TIME: 11 AM -1 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

QUESTION 1 (30 MARKS)

a) Explain the meaning of the following terms as used in ring theory.

   i) Euclidean Domain

   ii) Integral domain

   iii) Principal Ideal Domain

   iv) A Boolean ring (8 marks)

b) Determine whether or not the following polynomials are irreducible over Z_5

   i) $f(x) = x^3 + 2x^2 - 3x + 4$ (6 mks)

   ii) $g(x) = x^2 + 3x + 4$ (6 mks)

c) i) Show that a Boolean ring $\mathcal{B}, x^2 = x$ for each $x \epsilon \mathcal{B}$ implies $2x = 0$ (4 mks)

d) ii) Let x be a non zero element of a ring R with unity. Suppose there exists a unique $y \epsilon R$ such that $xyz = x$, show that $xy = 1 = yx$. (6 mks)

QUESTION TWO

a) Let R be a commutative ring with identity.

   i) Show that if e is an idempotent element of R, then $1 - e$ is also idempotent. (6 mks)

   ii) Show that if e is an idempotent element of R then $R \cong Re \oplus R(1 - e)$ (14 mks)

QUESTION THREE

a) Let R be the ring of real numbers with unity, and let $R[x]$ be the polynomial ring over R. Let $J = (x^2 + 1)$ be the ideal in $R[x]$ consisting of the multiples of $x^2 + 1$. Show that the quotient ${R[x]}/{J}$ is the field of complex numbers. (12 mks)

b) Let $f: R \to S$ be a homomorphism of the ring R into a ring S. Show that the set $\{f(a) | a \epsilon R\}$ is a subring of R (8 mks)

QUESTION FOUR

a) Find $q(r) and\ r(x)$ in $Z_5[x]$ if $g(x) = 2x^3 + 3x^2 + 4x + 1$ is divided by $f(x) = 3x + 1$. (8 mks)

b) Determine the idempotent, nilpotent elements and the units of the ring of integers modulo 10 (Z_{10}) (6 mks)

c) Find all cyclic subgroups of the group of units of the ring of integers modulo 24 (Z_{24}) (6 mks)

QUESTION FIVE

a) Show that the ring of Gaussian integers $R = \{m + n\sqrt{-1}\ | m, n \epsilon Z\}$ is a Euclidean ring if we set $\phi(m + n\sqrt{-1}) = m^2 + n^2$ (12 mks)

b) Let $A\ and\ B$ be ideals in R such that $B \subseteq A$. Show that $R / A \cong (R / B) / (A / B)$ (8 mks)