(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 442**

COURSE TITLE: **MULTIVARIATE ANALYSIS**

DATE: 18/02/2021 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1: (30 Marks) (COMPULSORY)

a) Define the following terms as used in multivariate statistics.

i. Multiple correlation [3mks]

ii. Wishart distribution [3mks]

iii. Hotelling T^2 distribution [3mks]

iv. Principal component [3mks]

b) Define the coefficient of skewness and Kurtosis for a multivariate vector with a normal distribution i.e. $x \sim N_P(\mu, \Sigma)$ [6mks]

c) Given that $x \sim N_P(\mu, \Sigma)$, show that $E(x-\mu)(x-\mu)' = \Sigma$ [6mks]

d) Show that the sample variance-covariance matrix S is given by $S = \frac{1}{n} X'HX$ where $H = I - \frac{1}{n} 11'$ [6mks]

QUESTION 2: (20 Marks)

a) Given the multivariate density function x with the distribution $x \sim Np(\mu, \Sigma)$, obtain its moment generating function [5mks]

b) Let the random variable $x \sim N_3(\mu, \Sigma)$ with $\mu' = (2 \quad -3 \quad 1)$ and $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$.

Find the distribution of $3x_1 - 2x_2 + x_3$ [6mks]

c) Show how a standardized P-dimensional normal variable y is obtained from a P-dimensional variable x with mean μ and covariance matrix Σ [4mks]

d) Suppose $x \sim N_2(\mu, \Sigma)$ where $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$

Obtain the first and second principal components [5mks]

QUESTION 3: (20 Marks)

a) State and explain briefly the three levels of measurements in a multivariate statistics **[6mks]**

b) Show that the mean of S is a bias estimator of Σ and hence give the unbiased estimate of Σ **[4mks]**

c) Prove that $f(x) = \frac{e^{-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)}}{|\Sigma|^{\frac{1}{2}}(2\bar{x})^{\frac{p}{2}}}$, where $-\infty < x_i < \infty$ and $-\infty < \mu_i < \infty$, $i = 1, 2, \ldots, p$ is a probability density function **[5mks]**

d) Suppose $X \sim Np(\mu, \Sigma)$, $\Sigma > 0$, find the characteristic function $(Q_X(t))$ of X **[5mks]**

QUESTION 4: (20 Marks)

a) Given that x has a trivariate normal distribution with mean $\mu' = (\mu_1\ \mu_2\ \mu_3)$ and $Var(x_i) = \sigma_i^2$ and $Cov(x_i, x_j) = \rho_{ij}\sigma_i\sigma_j \quad i \neq j = 1, 2, 3$
Show that the matrix of partial regression coefficient of x_1 given x_2 and x_3 is
$B_{12.3} = \frac{\sigma_1}{\sigma_2}\left(\frac{\rho_{12}-\rho_{13}\rho_{23}}{1-\rho_{23}^2}\right)$ **[5mks]**

b) Suppose $X \sim N_p(\mu, \Sigma)$, $\Sigma > 0$ and y is given by $y = ax$
Find the characteristic function of y and show that $y \sim N_p(a'\mu,\ a'\Sigma a)$ **[8mks]**

c) Suppose $X \sim N_p(\mu, \Sigma)$, $\Sigma > 0$. Show that $Q = (X-\mu)'\Sigma^{-1}(X-\mu)$ has a χ^2- square distribution with p degrees of freedom **[7mks]**

QUESTION 5: (20 Marks)

a) Show that if $V_1 \sim W_p(\Sigma, n_1)$ and $V_2 \sim W_p(\Sigma, n_2)$ then, $V_1 + V_2 \sim W_p(\Sigma, n_1 + n_2)$ if V_1 and V_2 are independent and hence give the general $V_1 + V_2 + \cdots + V_k$ Wishart distribution **[4mks]**

b) Let $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ have a multinomial distribution where $X_1 = x_1$ and $X_2 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

Let also $\mu = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}$ be the mean vector and $\Sigma = \begin{pmatrix} 7 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ be the variance-covariance matrix

compute

i. $E(X_1|X_2)$ **[4mks]**

ii. $Var(X_1|X_2)$ **[4mks]**

c) Given that in part (a) above, if $X_1 = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $X_2 = x_3$. Find

i. $E(X_1|X_2)$ **[4mks]**

ii. $Var(X_1|X_2)$ **[4mks]**