(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 406**

COURSE TITLE: FIELD THEORY

DATE: 11/02/2021 **TIME: 11 AM -1 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 (30 MARKS)

a) Define the following terms

i) Field (2 marks)

ii) Degree of $p(x) \in R[x]$ (2 marks)

iii) Characteristic of a field (3 marks)

b) Find the $G.C.D$ of $f(x) = 3x^3 + 5x^2 + 6x$ and

$g(x) = 4x^4 + 2x^3 + 6x^2 + 4x + 5$ in $Z_7[x]$ (9 marks)

c) State and prove the Remainder Theorem. (5 marks)

d) The algebraic element α is algebraic over F iff the simple extension $F(\alpha)/F$ is finite. Prove.

(e) Define a monic polynomial. (3 mks)

QUESTION TWO (20 MARKS)

a) Obtain the roots of $f(x) \in Q(x)$ with $f(x) = 2x^4 + x^3 - 21x^2 - 14x + 12$ (13 marks)

b) Show that $f(x) = x^4 + 4x^2 + x - 1$ is irreducible in $Q(x)$ (7 marks)

QUESTION THREE (20 MARKS)

a) Let F be a field then $f(x)$ is a unit in $F(x)$ iff $f(x)$ is a non-zero constant polynomial. Prove. (6 marks)

b) If $f(x)$ is a polynomial of degree two or three over a field F then $f(x)$ is irreducible over F iff $f(x)$ has no zeros in F. Prove (8 marks)

c) Determine whether the following is irreducible over Z_5

$f(x) = x^3 + 2x^2 - 3x + 4$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Define the following terms

i) A splitting field (3 marks)

ii) A normal extension of a field F (3 marks)

b) Find the splitting field for

$f(x) = x^4 - x^2 - 2$ in $Q(x)$ (6 marks)

c) What is the splitting field for $f(x) = x^4 + 4$ over Q (8 marks).

QUESTION FIVE (20 MARKS)

a) Define the following terms

i) Field extensions (3 marks)

ii) The degree of a field extension (3 marks)

b) If $p(x)$ is an irreducible polynomial over a field F, there exists an extension field of F that contains a zero of $p(x)$. Prove (8 marks)

c) If R is an integral domain and $p(x)$ and $q(x)$ are nonzero elements of $R(x)$, then $\deg(p(x)+q(x)) = \deg p(x) + \deg q(x)$. Prove (6 marks)