(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **STA 441**

COURSE TITLE: TIME SERIES ANALYSIS

DATE: 10/02/21 **TIME:** 11 AM -1 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION 1: (30 Marks) (COMPULSORY)

a) Explain the following terms as used in time series analysis:

i) Stationary process (1mk)
ii) Stationarity in the strong sense (1mk)
iii) Random walk process (1mk)
iv) Autoregressive process (1mk)
v) Purely random process (1mk)

b) i) Briefly describe the main objectives in the analysis of a time series. (3mks)

ii) State the unique feature that distinguishes time series from other branches of statistics. (1mk)

iii) Identify the main stages in setting up a Box-Jenkins forecasting model. (4mks)

c) Transform a time series $\{X_t\}$ into another series $\{Y_t\}$ where

$\boldsymbol{Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}}$ and $\boldsymbol{X_t = e^{i\lambda t}}$ and state the changes in its amplitude, wavelength and phase angle. (5mks)

d) Consider autoregressive process of order 1 (AR(1)) given by $X_t = \propto X_{t-1} + e_t$, where $\propto$ is a constant.

i) If $|\propto| < 1$, show that $X_t$may be expressed as infinite order of a MA process. (4mks)

ii) Find its autocovariance function $\left(\sigma(h)\right)$ and its autocorrelation function $\left(\rho(h)\right)$. (3mks)

e) Consider a moving average process given by $\mathrm{X_t = e_t + \beta e_{t-1}}$, where $(\beta_0 = 1,\ \beta_1 = 1)$.

Find its spectral density function. (5mks)

QUESTION 2: (20 Marks)

a) If an observed values $(X_1, X_2, \dots, X_n)$ on a discrete time series forms $n-1$ pairs of observation $(X_1, X_2), (X_2, X_3), \dots, (X_{n-1}, X_n)$ regarding the first observation in each pair as one variable and second observation as a second variable

Find:

i) The correlation coefficient ρ_1 between X_t and X_{t-1} (4mks)

ii) The correlation between observations at a distance k apart. (2mks)

b) Transform a moving average filter $\{X_t\}$ into another series $\{Y_t\}$ by the linear operator given that

$$\boldsymbol{X_t = e^{i\lambda t}} \text{ and } \boldsymbol{Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}}$$

Where

$$\boldsymbol{a_j = \begin{cases} \frac{1}{2m+1}, & j = 0, \mp 1, \mp 2, \dots\dots, \mp m \\ 0, & otherwise \end{cases}}$$ (14mks)

QUESTION 3: (20 Marks)

a) Consider an AR(1) process with mean μ given by

$X_t - \mu = \alpha(X_{t-1} - \mu) + e_t, t = 1, 2, 3, \dots\dots$

Find the estimates of the parameters α and μ using the method of least squares (8mks)

b) Show that the AR(2) process given $X_t = X_{t-1} - {}^1/_2 X_{t-2} + e_t$ is stationary and hence find its ACF. (14mks)

QUESTION 4: (20 Marks)

a) Find the autocovariance function ($\boldsymbol{\sigma(h)}$) and the autocorrelation function ($\boldsymbol{\rho(h)}$) of a moving average process of order q (MA(q)). (8mks)

b) Consider a second order process AR(2) given by
$X_t = {}^1/_3 X_{t-1} + {}^2/_9 X_{t-2} + e_t$.
Find if this process is stationary and hence if so obtain its ACF (12mks)

QUESTION 5: (20 Marks)

a) Suppose we have data up to time $\boldsymbol{n(x_1, x_2, \ldots, x_n)}$

i) Show that minimum mean squared error forecast of $\boldsymbol{x_{n+k}}$ is the conditional mean of $\boldsymbol{x_{n+k}}$ at time $\boldsymbol{n}$.
i.e. $\boldsymbol{\hat{x}(n,k) = E(x_{n+k}/x_1, x_2, \ldots, x_n)}$ (6mks)

ii) Consider the AR(1) model $\boldsymbol{X_t = \alpha X_{t-1} + e_t, \ |\alpha| < 1}$.
Forecast $\boldsymbol{x_{n+1}}$. (2mks)

b) The data below gives the average quarterly prices of a commodity for four (4) years.

Year	I	II	III	IV
1997	60.4	50.8	57.5	59.8
1998	48.3	43.6	63.2	79.5
1999	77.2	63.2	70.7	52.6
2000	65.1	66.4	71.6	75.1

Calculate the seasonal indices. (6mks)

c) Show that the spectral density function of an AR(1) process
$X_t = \alpha X_{t-1} + e_t$, given $|\alpha| < 1$ is given by $f(\lambda) = \frac{\sigma^2}{2\pi(1-2\alpha\cos\lambda+\alpha^2)}$
(6mks)