(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 403**

COURSE TITLE: **COMPLEX ANALYSIS I**

DATE: 16/7/2021 **TIME:** 9 AM - 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (Compulsory)

a) Define the following terms; (6 marks)

i. Laurent series,

ii. Harmonic conjugate,

iii. Singularity

iv. Define a Schwartz-Christoffel transformation

v. A conformal mapping $w = f(z)$ and hence state the condition the function is to satisfy for it to be conformal.

b) Find the residue of the following functions; (9 marks)

i. $f(z) = \frac{4-3z}{z^2-z}$

ii. $f(z) = \frac{e^z}{(z^2+1)z^2}$

iii. $f(z) = \frac{\sin z}{(z^2+z+1)\cos z}$

c) Show that $\oint_c \frac{\sin z}{z^4} dz = -\frac{\pi}{3} i$, where $c: \ |z| = 1$,described in a positive direction.

(5 marks)

d) Show that the function $\emptyset = x^3 - 3xy^2 + 2y$ can be a real part of analytic function.

Find the imaginary part of the analytic function. (5 marks)

e) Discuss the singularity of the following function: $f(z) = \frac{z \cos z}{(z-1)(z^2+1)^2(z^2+3z+2)}$

(5 marks)

QUESTION TWO

a) Expand the function $f(z) = \frac{1}{(z+1)(z+2)}$ in a Laurent series in the powers of $(z-1)$ valid in the annular domain containing the point $z = \frac{7}{2}$. (5 marks)

b) Let $f(z)$ be analytic inside and on a simple closed curve C except at a pole a of order m inside C. Prove that the residue of $f(z)$ at a is given by

$$a_{-1} = \lim_{z\to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \{(z-a)^m f(z)\}$$ (5 marks)

c) Find $I = \int_0^{2\pi} \frac{\cos 3\theta d\theta}{5-4\cos\theta}$ (10 marks)

QUESTION THREE

a) Evaluate $\int_{-\infty}^{\infty} \frac{z^2+3}{(z^2+1)(z^2+4)} dz$ (5 marks)

b) Find the residue of the function $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$. (5 marks)

c) Find a Schwartz-Christoffel transformation that maps the upper half plane H to the inside of a triangle vertices -1, 0 and i. (5 marks)

d) Evaluate $\int_0^{2\pi} \frac{d\theta}{3-2\cos\theta+\sin\theta}$ (5 marks)

QUESTION FOUR

a) expand $f(Z) = \frac{1}{(Z+1)(Z+3)}$ in a Laurent series valid for

i. $1 < |Z| < 3$ (4 marks)

ii. $|Z| < 3$ (2 marks)

iii. $0 < |Z + 1| < 2$ (2 marks)

iv. $|Z| < 1$ (2marks)

b) Evaluate $\int_{-\infty}^{\infty} \frac{z^2 dz}{(z^2+1)^2(z^2+2z+2)}$ (5 marks)

c) Determine the Laurent series of $f(z) = (z-3)\sin\frac{1}{z+2}$ (5 marks)

QUESTION FIVE

a) Find $I = \int_0^{2\pi} \frac{\cos 2\theta d\theta}{5-4\sin\theta}$ (5 marks)

b) Using residues, show that $\int_{-\infty}^{\infty} \frac{x^2+3}{(x^2+1)(x^2+4)} dx = \frac{5}{6}\pi$ (5 marks)

c) Consider the contour C defined by $x = y, x > 0$ and the contour C_1 defined by $x = 1, y \geq 1$. Maps these two curves using $w = \frac{1}{z}$ and verify that their angle of intersection is preserved in size and direction. (10 marks)