(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

YEAR ONE SEMESTER ONE EXAMINATIONS

FOR THE DEGREE OF MASTER OF SCIENCE PURE MATHEMATICS

COURSE CODE: MAT 812

COURSE TITLE: GROUP THEORY I

DATE: 16/9/2021 **TIME:** 9 AM - 12 NOON

INSTRUCTIONS TO CANDIDATES

Answer any Three Questions

TIME: 3 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over

QUESTION ONE (20 MARKS)

a. Define the following
   i. Group action (3 marks)
   ii. Kernel (1 marks)
   iii. Orbit (1marks)
   iv. Transitive (2 marks)

b. Suppose that a group G acts on a set X. Show that the size of the orbit is the index of the stabilizer .i.e. $[B(x)] = [G: Stab(x)]$ (8 marks)

c. Let the finite Group G act on the set X and denote by X^g The set of elements of X fixed by g. show that the number of orbits $= \frac{1}{G}\sum_{g\in G}|X^g|$ (5 marks)

QUESTION TWO (20 MARKS)

a. Define the following
   i. External Semi-Direct Product (2marks)
   ii. Internal semi-direct product (2marks)
   iii. Internal direct product (2 marks)

b. Let H and K be groups, and let $\rho: K \to Aut(H), k \to \rho k$ be a group homomorphism, show that the binary operation $(H \times K) \times (H \times K) \to (H \times K), ((h,k), (h^{/}, k^{/})) \to (h\rho_k(h^{/}, kk^{/})$ endows $H \times K$ with a group structure, with identity element (1,1). (10marks)

c. Let G be a group with subgroups H and K . Suppose that $G = HK$ and $H \cap K = (1_G)$.Then every element g of G can be written uniquely in the form hk for all $h \in H$ and $k \in K$. (4 marks).

QUESTION THREE (20 MARKS)

a. Define the following
   i. Solvable group (1marks)
   ii. Nilpotent group (1marks)
   iii. Central series (3 marks)
   iv. Nilpotency class (1mark)

b. If P is a non- trivial finite p- group, show that P has a trivial center (6 marks)

c. State the following theorems
   i. Correspondence theorem (3 marks)
   ii. Jordan holder theorem (3marks)
   iii. Fundamental theorem of arithmetic (3marks)

QUESTION FOUR (20 MARKS)

a. Define the following
   i. Subnormal series (3marks)
   ii. Composition series (3marks)
   iii. Central series (3marks)
   iv. Nilpotency class (1mark)

b. Show that every abelian group is solvable (2 marks)

c. Show that a group G which is both simple and solvable is cyclic of prime order (5marks)

d. Show that every finite abelian group and every finite p-group is nilpotent (3marks)

QUESTION FIVE (20 MARKS)

a. Define the following
   i. Free group (2marks)
   ii. Torsion subgroup (1mark)
   iii. Torsion free (1 mark)
   iv. Finitely generated group (2marks)

b. Show that if X is a basis for free group F then X generates F (4 marks)

c. Suppose F is a free group with a basis X and G is a free group with a basis Y. Show that $|F| \approx |G|$ (6marks)

d. Let G be an abelian group. Show that torsion subgroup of G is a subgroup of G. (4marks)