(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **STA 142**

COURSE TITLE: **INTRODUCTION TO PROBABILITY**

DATE: 20/07/21 **TIME:** **9 AM -11 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

1. (a) Distinguish between permutation and combinations counting techniques. (2 mks)

   (b) Suppose $A, B \in S$, show that $P(A \cap B') = P(A) + A \cap B$ (2 mks)

   (c) If a four digit number is formed from the digits 2,3,4 and 5 and repetitions are not allowed, find the probability that the number is not divisible by 5. (4 mks)

   (d) In how many ways can a first year student select 9 out of 18 courses :

      i. with no restrictions (2 mks)

      ii. if 6 courses are compulsory (3 mks)

   (e) Briefly explain any two types of random variables and give an example in each case (4 mks)

   (f) Let $P(A) = 0.75, P(A \cup B) = 0.625$ and $P(B') = 0.6$. Find $P(A \cap B)$ and $P(A' \cap B')$ (2 mks)

   (g) Consider the experiment of throwing a fair die. Let X be the random variable which assigns 1 if the number that appears is even and 0 if the number that appears is odd

      i. Find the range of X (1 mk)

      ii. Find the $P(X = 1)$ and $P(X = 0)$ (4 mks)

   (h) Suppose X is a random variable with p.d.f.

$$f(x) = \begin{cases} Kx^2, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

   where K is a constant

      i. Determine the value of K (2 mks)

      ii. Calculate $P(0 \leq x \leq \frac{1}{2})$ (3 mks)

   (i) For random variable X, the cumulative distribution for $F(x)$ is as shown

X	0	1	2	3	4	5	6
F(x)	0.03	0.07	0.13	0.25	0.65	0.8	1

      i. Find $P(X = 3)$ (2 mks)

      ii. Find $P(X > 2)$ (3 mks)

QUESTION TWO (20 MARKS)

2. (a) Let X be random variable with pdf

$$f(x) = \begin{cases} \frac{3}{39}, & 4 < x < 7 \\ 0, & elsewhere \end{cases}$$

Compute;

i. $P(X < 5)$, (2 mks)

ii. $P(5 < X < 6.5)$ (3 mks)

iii. $E(X + 2)^3$ (3 mks)

(b) Customers who purchase a certain make of a car can order an engine in any of the three sizes of all cars sold, 45% have the smallest engine, 35% have medium sized one, and 20% have the largest. Of cars with the smallest, 10% fail an emissions test within two years of purchase, while 12% of those with the medium size and 15% of those with the largest engine fail.

i. What is the probability that a randomly chosen car fail an emission test within two years? (6 mks)

ii. What is the probability that it is for a car with the small engine? (6 mks)

QUESTION THREE (20 MARKS)

3. (a) Two dice are thrown. If a 1 or a 6 turns up you are paid sh.20 but if neither a 1 or a 6 turns up, you will pay sh.50.

i. How much would you expect to lose in 9 games (6 mks)

ii. Find the minimum amount required to be paid to make the game worth while to yourself? (5 mks)

(b) Two urns each contain three beads. The first urn contains beads labeled 1, 3 and 5. The second urn contains beads labeled 2, 6, and 8. In a game, a player draws one bead at random from each urn and the score X, is the sum of the numbers on the two beads.

i. Obtain the six possible values of X and find their corresponding probabilities (2 mks)

ii. Calculate the standard deviation of X. (7 mks)

QUESTION FOUR (20 MARKS)

4. (a) Show that the following results hold for a random variable X
   i. $E(a) = a$ where a is a constant (2 mks)
   ii. $E(ax) = aE(x)$ where a is a constant (2 mks)

   (b) Let X (in tonnes) be a random variable representing the quantity of sugar sold in a day at a certain factory with a distribution function as shown;

$$P(X = x) = \begin{cases} C(2 - x), & X = 0, 1, 2 \\ C(x - 2), & X = 3 \\ 0, & elsewhere \end{cases}$$

where C is a constant.

   i. Find C such that $P(X = x)$ is a pdf (4 mks)
   ii. Sketch the probability bar graph and histogram of $P(X = x)$ (6 mks)
   iii. Compute the mean and variance of X (6 mks)

QUESTION FIVE (20 MARKS)

5. (a) State any three axioms of probability (3 mks)

   (b) The table below presents probabilities for the number of times that a certain computer system crashes in the course of a week. Let A be the event that there are more than two crashes during the week, and let B be the event that the system crashes at least once. Find a sample space that correspond to the events A and B then, find $P(A)$ and $P(B)$

NO. of crashes	0	1	2	3	4
Probability	0.60	0.30	0.05	0.04	0.01

   (c) A factory produces items in boxes of 2. Over the long run:
   92% of boxes contain 0 defective items;
   5% of boxes contain 1 defective item; and
   3% of boxes contain 2 defective items.
   A box is picked at random from production, then an item is picked at random from the box. Given that the item is defective, what is the chance that the second item in the box is defective? (7 mks)