(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAA 314

COURSE TITLE: METHODS I

DATE: 21/7/2021 **TIME: 9 AM – 11 AM**

INSTRUCTIONS

Answer **Questions ONE and** Any other **TWO**

TIME: 2 HOURS

QUESTION 0NE [30MKS]

a. Use special function to evaluate $\int_0^1 x^5(1-x)^6 dx$ 4mks

b. Suppose $f(t)=0$ for $t<0$ and that $a>0$, show that $L\{f(t-a)\}=e^{as}F(s)$ (4mks)

c. Prove that $J_0^{'}(x)=-J_1(x)$ 5mks

d. Using Rodrigues's formula derive the first four terms (5mks)

e. State whether the following functions are even or odd (3mks)

i. $f(x)=\sin x;\ \frac{-\pi}{2}<x<\frac{\pi}{2}$

ii. $f(x)=\cos x;\ \frac{-\pi}{2}<x<\frac{\pi}{2}$

f. Evaluate $\int_0^\infty x^3 e^{-4x}dx$ using special functions (4mks)

g. Evaluate the Bessel function $J_0(x)$ and $J_1(x)$ when x=1, correct to 3 decimal places (5mks)

QUESTION TWO [20MKS]

a. Let f(x) be a 2π-periodic function such that $f(x)=x^2$ for $x\in[-\pi,\ \pi]$. Find the Fourier series for the parabolic wave. (8mks)

b. Show that $L\{\cos at\}=\frac{s}{s^2+a^2}$ (5mks)

c. Solve the initial value problem $y'-5y=-e^{-2t}$, y(0)=3 (7mks)

QUESTION THREE [20MKS]

a. Prove that $\Gamma(1)=\Gamma(2)$ (3mks)

b. Prove that $B(m,n)=2\int_0^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta\, d\theta$ (3mks)

c. Express the polynomial $7x^4-3x+1$ in terms of Legendre series (6mks)

d. Solve the following differential equation by the method of power series

$y''-2xy'+y=0$ (8mks)

QUESTION FOUR [20MKS]

a. Give the definition of an ordinary point and state whether the following equations have an ordinary or singular point (3mks)

i. $x^2y''+(x^2+x)y'-y=0$

ii. $x^2y''+(1+2x)y'=0$

b. Solve $x^2y''+5xy'+(3-x)y=0$ using the method of Frobenius. (12mks)

c. evaluate $\int_0^\infty x^6e^{-2x}dx$ (5mks)

QUESTION FIVE [20MKS]

a. classify the following equation (3mks)

i. $2U_{xx}-4U_{xy}+2U_{yy}=0$

ii. $9U_{xx}+20U_{xy}+5U_{yy}=0$

b. Using direct integration to solve the equation $\frac{\partial^2 U}{\partial x^2}=\sin(x+y)$ given that at $y=0,\ \frac{\partial U}{\partial x}=1$ and at $x=0,\ U=(y-1)^2$ (7mks)

c. Use the method of separation of variables to solve $U_x-2U_t=U$ hence show that $U(x,\ 0)=6e^{-3t}$ (10mks)