(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

COURSE CODE: MAT 817

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 22/06/21 **TIME:** 9 AM -12 AM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

QUESTIONONE [20 MARKS]

(a) Define the following terms

(i) Conformal mapping (2 mks)

(ii) Analytic continuation (2 mks)

(b) If $f(z) = z^6 - 2z^5 + 3z + 2 - i$, evaluate

$\int_C \frac{f^I(z)}{f(z)} dz$ where C encloses all zeros of $f(z)$ (4 mks)

(c) Determine the linear fractional transformation that maps $z = i, 1, 0$ onto $w = 0, -i, -1$ respectively (6 mks)

(d) Evaluate $\oint_C \frac{\cos\pi z + \sin\pi z}{(z-1)(z-2)} dz$ (6 mks)

QUESTION TWO [20 MARKS]

(a) Find the residuals of the function $\boldsymbol{f(z) = \frac{z^3-2}{(z^2+9)^2}}$ (5 mks)

(b) Evaluate $\oint_C \frac{e^{2z}}{(z-1)^4} dz$ where C is a circle $|z| = 3$ (5 mks)

(c) Prove that $\oint 4z dz = 0$ (5 mks)

(d) Determine the number of zeros of $z^6 - 5z^2 + z - 2$ interior to $|z| = 1$ (5 mks)

QUESTION THREE [20 MARKS]

Consider the triangle $\boldsymbol{P(0,1)}$, $\boldsymbol{Q(1,1)}$ and $\boldsymbol{R(1,0)}$

(i) Draw the triangle and its image under $T(z) = z^2$ (10 mks)

(ii) Discuss conformity of T at $R(1,0)$ and $Q(1,1)$ (10 mks)

QUESTION FOUR [20 MARKS]

(a) State the Riemann mapping theorem (2 mks)

(b) Prove that the function $f_1(z) = \int_0^\infty 3t^3 e^{-zt} dt$ is analytic at all points of z for which $Re z > 0$ (5 mks)

(c) State and prove the Rouche's theorem (13 mks)

QUESTION FIVE [20 MARKS]

(a) Show that $\tanh^{-1} z = \frac{1}{2} \ln\left(\frac{1+z}{z-1}\right)$ (5 mks)

(b) Find the Laurent series about the indicated singularity for the function

$f(z) = \frac{e^{2z}}{(z-1)^3}$ $z = 1$ (4 mks)

(c) Evaluate $\int_i^{2-i} (3xy + iy^2) dz$

(i) Along the straight line joining $z = i$ and $z = 2 - i$ (5 mks)

(ii) Along the curves $x = 2t - 2$ and $y = 1 + t - t^2$ (6 mks)