(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAP 312

COURSE TITLE: LINEAR ALGEBRA III

DATE: 22/7/2021 **TIME:** 9 AM - 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

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QUESTION ONE (30 MARKS)

a) Define the following:

(i) Bilinear form over F. (2 marks)

(ii) B-Orthogonal compliment. (2 marks)

(iii) A quadratic function on V. (3 marks)

(iv) An isometry. (4 marks)

(v) Tensor product. (2 marks)

b) Show that matrix A= $\begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix}$ is a nilpotent matrix of index 2. (4 marks)

c) Outline any five applications of finite vector spaces. (5 marks)

d) Given that v = (1 +2i, 3-i) and u = (-2 +i, 4) are vectors in the complex vector space C^2, determine the vector 3v-(5-i) u. (4 marks)

e) Calculate the conjugate transpose of matrix A.

$$A = \begin{bmatrix} 1 & -2-i & 5 \\ 1+i & i & 4-2i \end{bmatrix}$$ (4 marks)

QUESTION TWO (20 MARKS)

a) Define the following:

(i) Eigenvalue and eigenvector. (3 marks)

(ii) Determinant of a matrix. (2 marks)

(iii) Trace of a matrix. (2 marks)

b) Given the following 2 × 2 matrix A = $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$,

Find its eigen equation. (10 marks)

c) Given that A is a real matrix with a complex eigenvalue $\lambda = \mu + i\nu$ and corresponding eigenvector $\nu = x + iy$, find the complex conjugate λ and the complex conjugate eigenvector. (3 marks)

QUESTION THREE (20 MARKS)

a) Show that if matrix A is Hermitian, then all the eigenvalues of A are real. (10 marks)

b) Show that if matrix A is Hermitian, then any two eigenvectors from different eigenspaces are orthogonal in the standard inner product for $\mathbb{C}^n$, ($\mathbb{R}^n$, If A is real symmetric). (10 marks)

QUESTION FOUR (20 MARKS)

a) Define the terms:
   (i) Hermitian matrix. (2 marks)
   (ii) Symmetric matrix. (2 marks)
   (iii) Nilpotent matrix. (2 marks)

b) Show if the matrix A = $\begin{bmatrix} 2 & -1 \\ 1 & -2 \end{bmatrix}$ of index 2 is nilpotent or not. (4 marks)

c) Show that S = $\{(i,0,0) \quad ,(i,i,0) \quad ,(0,0,i)\}$
   where $v_{1=} (i, \quad 0, \quad 0)$
   $v_{2=} (i, \quad i, \quad 0)$
   $v_{3=} (0, \quad 0, \quad i)$
   is a basis for C^3. (10 marks)

QUESTION FIVE (20 MARKS)

a) Define the terms:
   (i) Jordan block. (2 marks)
   (ii) Jordan form. (2 marks)
   (iii) Jordan chain. (2 marks)

b) Determine the Jordan form of the operator represented by the matrix

$$A=\begin{pmatrix} 1 & -1 & 0 \\ -1 & 4 & -1 \\ -4 & 13 & -3 \end{pmatrix}$$ (7 marks)

c) Show that if (V, q) is a quadratic form over F = F_2 and that dim V ≥ 4 , there exists a Vector v$\in V$ with q (v) = 0. (7 marks)