(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 401**

COURSE TITLE: **TOPOLOGY I**

DATE: 13/7/2021 **TIME:** 2 PM - 4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

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UNIVERSITY EXAMINATIONS, 2020/2021 ACADEMIC YEAR

MAIN EXAMINATION, FOR THE DEGREE OF BACHELOR OF SCIENCE & EDUCATION SCIENCE

JULY 2021 COURSE CODE: MAT 401 COURSE TITLE: TOPOLOGY I

INSTRUCTIONS

Answer QUESTION ONE and any other TWO questions.

QUESTION ONE (30marks)

a). Define the following terms

i). A metric on a set (3 Marks)

ii). A limit point of a topological space (2 Marks)

iii). A quotient space (2 Marks)

iv). A homeomorphism (2 Marks)

b). Let τ_1 and τ_2 be two topologies on $X = \mathbb{R}$. Define τ_1 and τ_2 suitably so that τ_2 is finer than τ_1. Show that, actually, τ_2 is finer than τ_1. (5 Marks)

c). Let (X, τ_f) a topological space where $\tau_f = \{U: U \subseteq X \mid X - U\ is\ finite$. or all of $X\}$ Show that τ_f is a topology. (6 Marks)

e). Find the interior of a set $B = (-2,0] \cup \{1\}$. (2 Marks)

f). Let X and Y be topological spaces and $f: X \to Y$ a function. Show that if for every closed set $B \in Y$ the set $f^{-1}(B)$ is closed in X then f is continuous. (4 Marks)

g). Prove that a sequence of points of a Hausdorff space converge to at most one point of X.

(4 Marks)

QUESTION TWO (20marks)

a). What do you understand by a metric topology? (2 Marks)

b). Consider a metric the usual d on $\mathbb{R}$ and set $A = (2,5) \subset \mathbb{R}$

(i). Show that A is bounded with respect to the metric d (2 Marks) (ii). Find the diameter of A (2 Marks)

(iii). Show that $2 \in \mathbb{R}$ is a limit point of A. (3 Marks)

(iv). Find the closure of A. (1 Marks)

c). Let X be a set. Prove that β a basis for a topology τ on X if an only if that τ equals the collection of all union of elements on β. (4 Marks)

d). (i). What do you understand by the term the basis of a topology (2 Marks)

ii). Describe two possible basis of the usual topology on $\mathbb{R}^2$. (4 Marks)

QUESTION THREE (20marks)

a). Let X and Y be two topological spaces. Define two projections π_1 (on X) and π_2 (on Y) from the product space $X \times Y$ to X and Y respectively. If $U \times V$ is a rectangle in $X \times Y$, such that U and V are open sets in X and Y respectively, express $U \times V$ in terms of the projections π_1 and π_2.

(4 Marks)

b). What is an accumulation point? Show that any point $x \in [-1,1]$ is an accumulation point of the set $[-1,1]$. . (4 Marks)

c). Let X be a topological space. Prove that the following conditions holds

i). ϕ and X are closed (2 Marks)

ii). Arbitrary intersection of closed sets are closed (5 Marks)

iii). Finite union of closed sets are closed (5 Marks)

QUESTION FOUR (20marks)

a). Let (X, τ) be a topological spaces and $Y \subseteq X$ such that $\tau_Y = \{Y \cap U : U \in \tau\}$. Prove that τ_Y is a topology on Y hence, describe the basis of that topology with respect to that of τ. (8 Marks)

b). Consider the set $Z = [0,1] \cup (2,4]$

(i). Show that $[0,1]$ and $(2,4]$ are both closed and open in Z. (8 Marks)

(ii). Does Z have an isolation point? Explain (2 Marks)

(iii). Find the closure of $(2,4]$ in Z. (2 Marks)

QUESTION FIVE (20marks)

a). Consider the set $X = \{a, b, c\}$ and a topology on X defined by $\tau_X = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$.

(i). Prove that τ_X is a topology on X. (8 marks)

(ii). Find the interior of $A = \{a, b\}$. (3 marks)

(iii). Find all the limit points of the set $A = \{a, b\}$. (3 marks)

b). Let X be a metrizable topological space and $A \subset X$. Prove that if there is a sequence of points of A converging to x then $x \in \bar{A}$ $(closure\ of\ A)$ and the converse is true. (6 marks)