(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **STA 210**

COURSE TITLE: **PROBABILITY AND STATISTICS**

DATE: 05/02/2021 **TIME: 11 AM -1 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Explain the meaning of the following terms

(i) Moment generating function of random variable (2 mark)

(ii) Characteristics function of random variable (2marks)

b) If X is a discrete random variable and b is constant.

(i) Showthat $E(bX) = bE(X)$ (2 marks)

(ii) Suppose the probability mass function of X is

X	0	1	2	3
$f(x)$	0.2	0.1	0.4	0.3

Find

(I) $E(2)$ (1 mark)

(II) $E(X)$ (2 marks)

(III) $E(2X)$ (1 mark)

(IV) $E(X^2)$ (2 marks)

(V) $E(2x + 3X^2)$ (1 marks)

c) A random variable X has pdf

$$f(x) = \begin{cases} \theta x, & 0 \leq x \leq 1 \\ 0 & elsewhere \end{cases}$$

(i) Find ? (2marks)

(ii) Obtain the distribution functions $F(x)$ and give its sketch (3 marks)

(iii) Find the variance of X? (3 marks)

(iv) Workout $p\left(X \leq \frac{1}{2}\right)$ (2 marks)

d) The total cost X of completing a project is assumed to follow a normal distribution with mean $850,000 and a standard deviation of $170,000. The revenue, R, promised to the contractor is $1,000,000.

(i) The contract will be profitable if revenue exceeds total cost. What is the probability that the contract will be profitable to the contractor? (2 marks)

(ii) Suppose the contractor has the opportunity to renegotiate the contract. What value of R should the contractor strive for in order to have a 0.99 probability of making profit? (3 marks)

e) If $X \sim B(50, 0.35)$. Find

(i) Probability of failure. (1 mark)

(ii) The number of experimental outcomes providing exactly one success in 50 trials. (1 marks)

QUESTION TWO (20 MARKS)

a) A random variable X is known to have a distribution with probability density function

$$f(x) = \begin{cases} 8x^{\alpha}, & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

(i) Which type of random variable is X? (1 mark)

(ii) Find the constant α (2 marks)

(iii) What is the variance of X? (5 marks)

b) A random variable X follows a Binomial distribution.

(i) Give the pdf of X (1 mark)

(ii) Obtain the mgf of the distribution of X and hence compute the mean (8 marks)

c) It is known that all items produced by a certain machine will be defective with probability 0.1, independently of each other. What is the probability that in a sample of three items, at most one will be defective? (3 marks)

QUESTION THREE (20 MARKS)

a) A random variable X has a poison distribution such that

$$p(x = 2) = \frac{2}{3}p(x = 1).$$

Find

(i) $p(x = 0)$ (4 marks)

(ii) Find the moment generating function of X (4 marks)

b) Given that $E(5 + X) = 15$ and $E(5 + X)^2 = 226$, determine

(i) $Var(5 + X)$ (1 marks)

(ii) $E(X)$ (2 marks)

(iii) $Var(2X)$ (3 marks)

c) If X equal the birth weight (in grams) of babies in the Singapore and assuming the distribution of X is $N(14, 2.5^2)$, find

(i) $p(X \ge 18)$ (2 marks)

(ii) $P(X \le 8)$ (2marks)

(iii) $P(12 \le X \le 15)$ (2 marks)

QUESTION FOUR (20 MARKS)

a) If a random variable of X has pdf

$$f(x) = \begin{cases} cx^2, & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

(i) Find c (2 marks)

(ii) Obtain of the mean of X (3 marks)

(iii) Find the variance of X (3 marks)

(iv) Find the cumulative distribution function of X (2 marks)

b) Identify the following variables as either discrete or continuous

(i) Distance from school to home (1 mark)
(ii) Own Cow or Sheep (1 mark)
(iii) Recorded values of temperature of a place (1 mark)

c) A random variable X has the moment generating function

$$M_X(t) = \frac{1}{3} + \frac{2}{3}e^t$$

(i) State the probability distribution of X? (2marks)
(ii) Show that $M_X'(0) - [M_X''(0)]^2 = Var(X)$ (5 marks)

QUESTION FIVE (20 MARKS)

a) If X is a discrete random variable with pdf $f(x)$ and a is a constant, show that $E(7 + aX) = 7 + aE(X)$ (3 marks)

b) If a random variable X has cumulative distribution function given as

$$F(x) = -e^{-\beta x} + 1, 0 < x < \infty, \beta > 0$$

(i) Sketch $F(x)$ (2 marks)
(ii) Obtain the pdf of X (2 marks)
(iii) Identify the distribution of X (1 mark)
(iv) Obtain moment generating function of X and hence find the mean and Variance of X (10 marks)

c) Suppose that the probability of female birth is 0.3. If 10 individuals are selected in this population. What is the probability of getting 6 women? (3 marks)