(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 321/MAA 311

COURSE TITLE: ODE I

DATE: 20/07/2021 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) State the order and degree of the following differential equations

i) $x\left(\frac{dy}{dx}\right)^2 - 4y^3 = e^{2x}$ (2 mks)

ii) $y^{III} + 3(y^{II})^3 = 5y$ (2 mks)

b) Prove that it is homogeneous and solve the differential equation

$(3x^2 - y^2)dx + (xy - x^3y^{-1})dy = 0$ (6 mks)

c) Solve the linear fractional differential equation

$(2x + y + 6)dx = -(x - 2y - 2)dy$ (7 mks)

d) Solve the non-homogeneous differential equation

$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 3sinx$ (6 mks)

e) A car engine temperature by the time it is shut off is 180^0C. The surrounding air temperature is 35^0C. After 15 seconds have elapsed, the engine temperature is 150^0C.

(i) How long will it take the engine temperature to cool to 50^0C (5 mks)

(ii) Find the engine temperature after 20 seconds (2 mks)

QUESTION TWO [20 MARKS]

(a) Obtain the differential equation having a solution as

$y = Ae^{-2x} - Be^{3x} + C$ (7 mks)

(b) Solve the differential equations using appropriate method

(i) $2ydx + x^2dy = 0, y(0) = -2$ (5 mks)

(ii) $(2xy + 3)dx + (x^2 - 1)dy = 0$ (8 mks)

QUESTION THREE [20 MARKS]

(a) The sum of Kshs 2500 is invested at a rate of 6% per annum compounded continuously. What will be the amount after 6 years? (4 mks)

(b) Using the integrating factor solve the differential equation

$(x^2y - x)dy + (y + 2x^2)dx = 0$ (8 mks)

(c) Solve the following Bernoulli's equation

$x^2\frac{dy}{dx} - 2xy = 3y^4; y(1) = 1$ (8 mks)

QUESTION FOUR [20 MARKS]

(a) Verify that the functions $y_1 = x^2 + 2x + 2$ and $y_2 = e^x$ are linearly independent solutions of the differential equation $x\frac{d^2y}{dx^2} - (2 + x)\frac{dy}{dx} + 2y = 0$ (7 mks)

(b) Solve the differential equation $x^2dy + (y^2 - xy)dx = 0$ (5 mks)

(c) Use the method of undetermined coefficients to solve

$2y^{II} - 4y^{I} - 6y = 3e^{2x}$ (8 mks)

QUESTION FIVE [20 MARKS]

(a) Solve the differential equation

$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$ (6 mks)

(b) Solve the equation by method of variation of parameter

$y^{II} + 4y^{I} + 4y = x^2e^{-2x}$ (7 mks)

(c) Test for exactness and solve the differential equation

$(6y^2 - x^2 + 3)dy + (3x^2 - 2xy + 2)dx = 0$ (7 mks)