(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR,

THIRD YEAR SPECIAL/ SUPPLIMENTARY EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: STA 341

COURSE TITLE: THEORY OF ESTIMATION

DATE: 05/02/2021 **TIME:** 8 AM – 10 AM

INSTRUCTIONS TO CANDIDATES

Answer **Questions ONE** and **ANY OTHER TWO**

QUESTION ONE [30 MARKS]

a) Define the following terms
   (i) Consistency (2mks)
   (ii) Mean squared Error (2mks)

b) Suppose $x_1, x_2, x_3, \dots \dots \dots \dots x_n$ be a random sample from the Poisson distribution with mean λ. Show that the statistic $T = \sum_{i=1}^{n} x_i$ is sufficient for λ.

c) Let $x_1, x_2, x_3, \dots \dots \dots \dots x_n$ form a random sample of size n from a Bernoulli distribution with parameter θ. If $d(\underline{x})$ is unbiased estimator of θ. Find
   (i) Fisher's information $I(\theta)$. (7mks)
   (ii) Gramer-Rao lower bound for $d(\underline{x})$. (3mks
   (iii) Uniformly minimum variance unbiased estimator of θ (2mks)

d) Let $f(x;\theta) = \frac{2}{\theta^2}(\theta - x)$, $0 < x < \theta$. Let $x1, x2, x3, \dots \dots \dots \dots \dots xn$ be a random sample from X. Obtain the estimate of θ by method of moments (8mks)

QUESTION TWO (20 MARKS)

a) If $X \sim N(\mu, \sigma^2)$. Let $x_1, x_2, x_3, \dots \dots \dots \dots x_n$ be a random sample from the population. Find the maximum likelihood estimators of $\sigma^2$when μ is known. (13mks)

b) Suppose $x_1, x_2, x_3, \dots \dots \dots \dots x_n$ form a random sample of size n from a normal population having mean μ and σ^2, let $\bar{x}_n = \frac{1}{n}\sum_{i=0}^{n} xi$ and $S_n = \frac{1}{n-1}\sum_{i=0}^{n}(xi - \bar{x})^2$ be the sample mean and variance respectively. Show that S_n is unbiased estimator for σ^2. (7mks)

QUESTION THREE (20 MARKS)

a) $x_1, x_2, x_3, \dots \dots \dots \dots x_n$ is random sample from a population given by

$$f(x) = \begin{cases} 1, & \lambda - \frac{1}{2} < x < \lambda + \frac{1}{2} \\ 0, & elsewhere \end{cases}$$

Prove that $\bar{x}$ is a consistent estimator for λ (12mks)

b) Let $x_1, x_2, x_3, \dots \dots \dots \dots x_n$ denote a random sample from a Bernoulli distribution pdf

$f(x;\theta) = \theta^x(1-\theta)^{1-x}$ for $x = 0,1$. Show that the family of Bernoulli distribution belongs to one parameter exponential family hence determine a sufficient statistic for θ. (8mks)

QUESTION FOUR (20 MARKS)

(a) Let X have pdf $f(x;\theta) = (1-\theta)\theta^x$ for $x = 0, 1, 2, \dots \dots \dots$

Let $x_1, x_2, x_3, \dots \dots \dots \dots x_n$ be a random sample, take $\varphi(\theta) = \frac{\theta}{1-\theta}$. Find
   (i) Fisher's information $I(\theta)$. (7mks)
   (ii) Gramer-Rao lower bound for $d(\underline{x})$. (3mks

(iii) Minimum variance best unbiased estimator of θ (4mks)

(b) Suppose that $x_1, x_2, x_3, \dots \dots \dots \dots \dots x_n$ form a random sample of size n from a Bernoulli distribution with parameter θ. Show that the statistic $T = \sum_{i=1}^{n} x_i$ is sufficient for θ. (6mks)

QUESTION FIVE (20 MARKS)

a) Students in STA 341 class claimed that doing there assignment had not helped them prepare for the main exam. The exam score Y and assignment score X for 4 students were as follows

X	Y	XY	X^2
96	95	9120	9216
77	80	6120	5929
78	79	6162	6064
64	79	5056	9409

Obtain $\hat{\beta}_0$ and $\hat{\beta}_1$ and write prediction equation (10mks)

b) Let $x_1, x_2, x_3, \dots \dots \dots \dots \dots x_n$ be a random sample from the Poisson distribution, take $\varphi(\theta) = e^{-\theta}$.Find Gramer-Rao lower bound for $d(\underline{x})$. (10mks)