KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2019/2020 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER
SPECIAL/SUPLEMETARY EXAMINATIONS

FOR THE DEGREE OF B.ED (SCIENCE)& BSC (PHYSICS)

COURSE CODE: SPH 313

COURSE TITLE: QUANTUM MECHANICS I

DATE: 1/2/21 **TIME:** 11 – 1 Pm

INSTRUCTIONS TO CANDIDATES

TIME: 2 HOURS

Answer question ONE and any TWO of the remaining

KIBU observes ZERO tolerance to examination cheating

QUESTION ONE [30 MARKS]

a) Show that $[\hat{x}, \hat{p}_y] = 0$ [3 marks]

b) An electron with energy of $1.0eV$ is incident on a barrier $10.0eV$ high and $0.50nm$ wide. Find its transmission probability [$\hbar$=$1.054x10^{-34}Js$]. [4 marks]

c) Find the flux $J_x = \frac{\hbar}{2im}\left[\psi^* \frac{d\psi}{dx} - \left(\frac{d\psi^*}{dx}\right)\psi\right]$ associated with the wave function $\psi(x) = Ae^{ikx} + Be^{-ikx}$. [5 marks]

d) Find the expectation value $\langle x \rangle$ for the states $n = 0$ and $n = 1$ of a simple harmonic oscillator. [5 marks]

e) Give any two conditions satisfied by a well behaved wave function. [2 marks]

f) A particle is described by the wave function $\psi(x) = \begin{cases} 0, & x < 0 \\ \sqrt{2}e^{-x/L}, & x \geq 0 \end{cases}$ where $L = 1nm$. Calculate the probability of finding the particle in region $x \geq 1nm$. [4 marks]

g) Show that the zero point energy of a harmonic oscillator could not be lower than $\hbar\omega/2$ without violating the uncertainty principle. [5 marks]

h) What is the parity of 2p and 3d states of a hydrogen atom? [2 marks]

QUESTION TWO [20 MARKS]

a) In the ground state of a hydrogen atom show that the probability P for the electron to lie with within a sphere of radius R is: $P = 1 - exp\left(\frac{-2R}{a_0}\right)\left(1 + \frac{2R}{a_0} + \frac{2R^2}{a_0^2}\right)$ given that:- $\psi_{100} = (\pi a_0^3)^{-1/2} exp(-r/a_0)$. [8 marks]

b) Show that in spherical polar coordinates: $\frac{L^2}{(i\hbar)^2} = \frac{\partial^2}{\partial\theta^2} + \left(\frac{1}{sin^2\theta}\right)\frac{\partial^2}{\partial\varphi^2} + cot\theta\,\frac{\partial}{\partial\theta}$. [12 marks]

QUESTION THREE [20 MARKS]

a) Show that for a simple harmonic oscillator $\Delta x \Delta p_x = \hbar(n + \frac{1}{2})$ and its in agreement with uncertainty principle. [12 marks]

b) Prove that the Eigen value of the Hermitian operator is real. [8 marks]

QUESTION FOUR [20 MARKS]

a) The spin wave functions of two electrons is: $\frac{(x\uparrow x\downarrow - x\downarrow x\uparrow)}{\sqrt{2}}$. What is the Eigen value of $S_1.S_2$? [5 marks]

b) Show that for a neutron-proton system; $\sigma_p\sigma_n = \begin{cases} -3, & for\ singlet\ state \\ 1, & for\ triplet\ state \end{cases}$ [7 marks]

c) Find the expectation values of kinetic energy, potential energy and total energy of hydrogen atom in ground state for $\psi_0 = \frac{e^{-r/a_0}}{\sqrt{\pi a_0^3}}$ where a_0 is the Bohr's radius. [8 marks]

QUESTION FIVE [20 MARKS]

a) Consider the Hamiltonian for a one dimensional system of two particles of masses m_1 and m_2 subjected to potential that depends only on the distances between the particles $x = x_1 - x_2$ given by: $\hat{H} = \frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + V(x_1 - x_2)$. Write down the Schrödinger equation using new variables x and X where $x = x_1 - x_2$ and $X = \frac{m_1x_1+m_2x_2}{m_1+m_2}$. [12 marks]

b) Using separation of variables find equation governing the evolution of centre of mass and the relative distance of the particles. Interpret your results. [8 marks]