(Knowledge for Development)

KIBABII UNIVERSITY

MAIN EXAMINATION

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

FOR THE DEGREE OF BACHELOR OF EDUCATION

MATHEMATICS

COURSE CODE: **MAP 212**

COURSE TITLE: REAL ANALYSIS I

DATE: 18/02/2021 **TIME: 2 PM- 4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION 1 (30 MARKS)

(a) Explain what is meant by saying that a number r is rational. **(2mks)**

(b) Let $\{A_i : i = 1,2..\}$ be a collection of countable sets. Show that $A = \bigcup_i A_i$ is countable. **(3mks)**

(c) Give the definition of neighbourhood of a set **(2mks)**

(d) Define a subsequence. **(2mks)**

(e) Let (x_n) be a sequence of real numbers. Prove that$x_n \to x$ if and only if $\overline{lim}\, x_n = \lim inf x_n = x$. **(7mks)**

(f) State and prove the intermediate value theorem. **(10mks)**

(g) Giving one example, define a partition of an interval. **(4mks)**

QUESTION 2 (20 MARKS)

(a) Show that the set of real numbers is a metric space. **(10mks)**

(b) Let $A \subseteq \mathbb{R}$. Prove that A is closed if and only if A^c is open. **(6mks)**

(c) Let $B \subseteq \mathbb{R}$ be given by $B = \{x \in \mathbb{R}: 2 < x \leq 4\}$. Show that B is neither open nor closed. **(4mks)**

QUESTION 3 (20 MARKS)

(a) Let $x, y, z \in \mathbb{R}$. Show that $|x - y| + |y - z| \geq |x - z|$. **(4mks)**

(b) Let $A, B, C \subset X$. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **(10mks)**

(c) Prove that $\sqrt{7}$ is irrational. **(6mks)**

QUESTION 4 (20 MARKS)

(a) Give the definition of limit of a function and prove that this limit is unique. **(6mks)**

(b) Define the concept of uniform continuity. **(2mks)**

(c) Let f: $\mathbb{R} \to$R be defined by f(x) = $|x|$ $\forall\, x \in \mathbb{R}$. Show that f is continuous on $\mathbb{R}$. **(7mks)**

(d) Let $f(x) \begin{cases} 3 - x, & x > x \\ 1, & x = 1 \\ 2x, & x < 1 \end{cases}$ Sketch the graph of f and find f(1^+) and f(1^-) **(5mks)**

QUESTION 5 (20 MARKS)

(a) Let (x_n) be a sequence of real numbers. Prove that if $x_n \to$ x then $|x_n| \to |x|$. **(6mks)**

(b) Let (x_n) be a monotonic sequence of real numbers. Prove that (x_n) is convergent iff it is bounded. **(10mks)**

(c) Let (x_n) be a sequence of real numbers. Prove that if (x_n) is convergent then it is Cauchy .**(4mks)**