(Knowledge for Development)

KIBABII UNIVERSITY

MAIN EXAMINATION

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 204**

COURSE TITLE: REAL ANALYSIS I

DATE: 18/02/2021 **TIME: 2 PM- 4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms

   i. Disjoint sets (2marks)

   ii. Ordered field (2marks)

   iii. Bounded set (2marks)

   iv. Equivalence relation (3marks)

b) Prove that for some $n \in \mathbb{N}$, $\sum_{k=1}^{n} k^3 = \frac{1}{4} n^2(n+1)^2$ (6marks)

c) Show that $|a| + |b| \geq |a + b|$ (4marks)

d) Prove that a countable union of countable sets is countable (5marks)

e) Let A, B and C be sets. Show that

   i. $A(B \cup C) = (A \backslash B) \cap (A \backslash C)$ (3marks)

   ii. $A(B \cap C) = (A \backslash B) \cup (A \backslash C)$ (3marks)

QUESTION TWO (20 MARKS)

a) Let $\mathbb{F}$ be a field and $x, y \in \mathbb{F}$. Show that $|x| - |y| \leq |x - y|$. (4marks)

b) Show that the power set $P(\mathbb{N})$ of $\mathbb{N}$ is countable (5marks)

c) Define a function $f: \mathbb{N} \to \mathbb{Z}$ as $(n) = \begin{cases} \frac{n+1}{2} & where\ n\ is\ odd \\ 1 - \frac{n}{2} & where\ n\ is\ even \end{cases}$. Show that f is a bijection (6marks)

d) (i)Define the term Cartesian product of sets X and Y. (2marks)

   (ii)Given that $X = \{0,1\}$ and $Y = \{-1,0,2\}$, find the Cartesian product of X and Y. (3marks)

QUESTION THREE (20 MARKS)

a) Define the following terms

   i. Complete ordered field (2marks)

   ii. Supremum (2marks)

   iii. Infimum (2marks)

b) Let $\mathbb{F}$ be an ordered field. Define a metric d on the field as $d(x, y) = |x - y|$ for $x, y \in \mathbb{F}$. Show d is a metric. (6marks)

c) Find the infimum, supremum, minimum and maximum of the following sets.

   i. $A = \left(-1, \frac{1}{n}\right), n \in \mathbb{N}$ (4marks)

   ii. $B = \left[\frac{1}{n}, \frac{2+n}{n}\right], n \in \mathbb{N}$ (4marks)

QUESTION FOUR (20 MARKS)

a) State the completeness axiom (2marks)

b) Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x^2 + 1$ and $g(x) = x^3 - 2x - 3$. Find $(g \circ f)(-2)$ (5marks)

c) Let $n \in \mathbb{N}$. Let $\sim$ be a relation on $\mathbb{N}$ be defined as $x \sim y$ if $x \equiv y \operatorname{mod}(n)$,that is $x - y$ is divisible by n. Show that $\sim$ is an equivalence relation. (6marks)

d) Differentiate between injective and subjective functions giving examples in each case. (4marks)

e) If $\mathbb{F}$ is an ordered field and $A \subset \mathbb{F}$ is non empty then A has at most one least upper bound and at most one least lower bound. Proof (3marks)

QUESTION FIVE (20 MARKS)

a) Let $A$and B be two finite sets. Show that $(A \cap B)^c = A^c \cup B^c$ (5marks)

b) Prove that there is no rational number x such that $x^2 = 2$. (6marks)

c) Let $\mathbb{F}$ be an ordered field and $a \in \mathbb{F}, a \neq 0 \; iff \; a^2 > 0$ (4marks)

d) Show that for $n \geq 1$, $8^n - 3^n$ is divisible by 5 for $n \in \mathbb{N}$. (5marks)