(Knowledge for Development)

KIBABII UNIVERSITY

MAIN EXAMINATION

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAP 222**

COURSE TITLE: REAL ANALYSIS I

DATE: 18/02/2021 **TIME: 2 PM- 4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30MARKS)

a. Define the following

   i. Metric space (4marks)

   ii. Open set (2marks)

b. If $\{E_1, E_2, \ldots., E_n\}$ is any finite collection of closed subsets of X w.r.t.(X,d), show that $\bigcap_{i=1}^{n} E_i$ is also closed (5marks)

c. Let $a, b \in \mathbb{R}$ such that $a \leq b + \varepsilon$ for every $\varepsilon > 0$. Show that $a \leq b$ (5marks)

d. For sets A,B and C, Show that A ∪(B ∪ C) = (A ∪ B) ∪ C (6marks)

e. For any two finite sets A and B, show that $(A \cup B)^C = A^C \sqcap B^C$ (8 marks)

QUESTION TWO (20MARKS)

a. Define the following sets

   i. Bounded below (2marks)

   ii. Bounded above (2 marks)

   iii. Infimum (2marks)

b. Show that if x and y are positive real numbers, then x< y iff $x^2 < y^2$ (9marks)

c. Let S be a non-empty set of real numbers with sup say b. show that $\forall\ a < b\ \exists\ x \in S$ such that $a < x \leq b$ (5marks)

QUESTION THREE (20MARKS)

a. Define the following

   i. Surjective (2marks)

   ii. Injective (2marks)

b. Consider the function $f(1,-\infty) \rightarrow (0,1)$ defined by $f(x) = \frac{x-1}{x+1}$. Show that f posses an inverse $f^{-1}(y) = \frac{y+1}{1-y}$ (5marks)

c. Show that the empty set $\emptyset$ is always open (5marks)

d. Let A, B and C be sets. Show that $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ (6marks)

QUESTION FOUR (20MARKS)

a. Define the following

   i. Composition of a function (2marks)

   ii. Inverse of a function (2marks)

   iii. Cardinality (2marks)

b. Show that a set S of real numbers is bounded iff $\exists k \in \mathbb{R}$ such that $|x| \leq k$ for all $x \in S$ (8marks)

c. Show that $f(x) = \begin{Bmatrix} \frac{2x-6}{x-3} & when\ x \neq 3 \\ 2 & when\ x = 3 \end{Bmatrix}$ is continuous at $x = 3$ (6marks)

QUESTION FIVE (20MARKS)

a. Define the following

   i. Removable discontinuity (2marks)

   ii. Infinite discontinuity (2marks)

   iii. Finite discontinuity (2marks)

b. Show that $\emptyset, X$ are always closed in (X, d) (3marks)

c. Show that if $x \in \mathbb{R}$ then $|xy| = |x||y|$ (4marks)

d. Show that an infinite subset of a countable set is countable (3marks)

e. Let (X, d) be a metric space. Show that the union of any arbitrary family of subsets open in (X, d) is open in (X, d). (4marks)