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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

MAIN EXAMINATION

2020/2021 ACADEMIC YEAR

THIRD YEAR, FIRST SEMESTER

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 351

COURSE TITLE: ENGINEERING MATHEMATICS III

DATE: 15/7/2021

TIME: 9 AM – 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question **One** and **Any** other **TWO** Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

- (a) Find, from first principles the laplace transform for the function

$$f(t) = e^{3t} \quad (6 \text{ marks})$$

- (b) Find the directional derivative for the surface $\phi(x, y, z) = x^2z + 2xy^2 + yz^2$

At the point $(1, -1, 1)$ in the direction of the vector $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$

(5 marks)

- (c) Evaluate $\lim_{x \rightarrow 3} \frac{x^3 - 10}{x - 3}$ (5 marks)

- (d) Express $f(x) = x$ in Fourier series on the interval $[-3, 3]$ (8 marks)

- (e) Evaluate the inverse Laplace transform for the function $f(s) = \frac{3s+5}{s^2+7}$

(6 marks)

QUESTION TWO (20 MARKS)

- (a) (i) Express $f(s) = \frac{1}{(s-1)(s-2)(s+4)}$ in partial fractions

(ii) Hence evaluate the inverse laplace transform for $f(s)$ (8 marks)

- (b) Use the laplace transform to solve the initial value problem

$$\frac{dy}{dt} - 2y = e^{5t} \text{ subject to } y(0) = 3 \quad (8 \text{ marks})$$

- (c) Evaluate using the tables the Laplace transform for

$$f(t) = \cos 3t \quad (4 \text{ marks})$$

QUESTION THREE (20 MKS)

- (a) Given $\mathbf{P} = x^2z\mathbf{i} + xy\mathbf{j} + y^2\mathbf{k}$ and $\mathbf{Q} = yz^2\mathbf{i} + xy\mathbf{j} + x^2z\mathbf{k}$, determine an

expression for $\text{grad}(\mathbf{P} \cdot \mathbf{Q})$ (8 marks)

- (b) If $\mathbf{F} = xy^2\mathbf{i} - 2yz\mathbf{j} + xyz\mathbf{k}$, determine at the point $(1, -1, 2)$ $\text{Curl } \mathbf{F}$

(8 marks)

- (c) Show whether the vectors \mathbf{A} , \mathbf{B} and \mathbf{C} are coplanar or not, if

$$\mathbf{A} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}, \quad \mathbf{B} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}, \quad \mathbf{C} = 3\mathbf{i} + \mathbf{j} - \mathbf{k} \quad (4 \text{ marks})$$

QUESTION FOUR (20 MARKS)

(a) Distinguish between an odd and an even function, giving one example for each (4 marks)

(b) Find the half-range Fourier sine series expansion for the function

$$f(x) = x$$

(8 marks)

(c) Find the Fourier series expansion for the function

$$f(x) = \begin{cases} 0 & -\pi < x < -\frac{\pi}{2} \\ 4 & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x < \pi \end{cases}$$

$$f(x + 2\pi) = f(x)$$

(8 marks)

QUESTION FIVE (20 MARKS)

(a) Find the total differential for the function

$$f(x) = ye^{x+y}$$

(6 marks)

(b) Evaluate $\lim_{x \rightarrow \infty} \frac{2x+7}{x+4}$

(6 marks)

(c) Find the domain of the function $f(x, y, z) = \frac{\sqrt{x+y+z-7}}{2x-1}$

(5 marks)

(d) Discuss the continuity of the function $f(x, y) = \frac{x-2y^3}{x^2+y^2}$

(3 marks)