



*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2019/2020 ACADEMIC YEAR**  
**SECOND YEAR SECOND SEMESTER**  
**MAIN EXAMINATION**

**FOR THE DEGREE OF BACHELOR OF EDUCATION AND  
BACHELOR OF SCIENCE**

**COURSE CODE: MAP 221 / MAT 202**

**COURSE TITLE: LINEAR ALGEBRA II**

**DATE: 05/02/2021**

**TIME: 8 AM -10 AM**

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

### QUESTION ONE (30 MARKS)

- a) State and prove the **three** properties of a Euclidean space (6 marks)
- b) Let  $u = (2, -1, 1)$  and  $v = (1, 1, 2)$ . Find  $\langle u, v \rangle$  and the angle between these vectors. (3 marks)
- c) Show that the usual basis of Euclidean space  $\mathbb{R}^3: E = \{ e_1 = (0, 1, 0), e_2 = (1, 0, 0) \text{ and } e_3 = (0, 0, 1) \}$  form an orthonormal set in  $\mathbb{R}^3$  with the Euclidean inner product. (9 marks)
- d) Let  $F: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be defined by  $F(x, y, z) = (2x - 3y + 4z, 5x - y + 2z, 4x + 7y)$ . Find the matrix of  $F$  relative to the standard basis of  $\mathbb{R}^3 E = \{ e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1) \}$  (10 marks)
- e) Find the eigen values of the following characteristic of a matrix (2 marks)

$$\begin{vmatrix} \lambda + 2 & 1 \\ -5 & \lambda - 2 \end{vmatrix} = 0$$

### QUESTION TWO (20 MARKS)

- a) Given a vector  $v = (a, b, c)$  in  $\mathbb{R}^3$
- i) Show that  $\cos \alpha = \frac{a}{\|v\|}$  (2 marks)
- ii) Find  $\cos \beta$  (2 marks)
- iii) Find  $\cos \gamma$  (2 marks)
- iv) Show that  $\frac{v}{\|v\|} = (\cos \alpha, \cos \beta, \cos \gamma)$  (2 marks)
- v) Show that  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$  (2 marks)
- b) Let  $V$  be a vector space,  $u \in V$  and  $\alpha$  is a scalar. Prove that the following properties hold.
- i)  $0u = 0$  (2 marks)
- ii)  $\alpha 0 = 0$  (2 marks)
- iii)  $(-1)u = -u$  (2 marks)
- iv) If  $\alpha u = 0$  then  $\alpha = 0$  or  $u = 0$  (4 marks)

### QUESTION THREE (20 MARKS)

- a) Let  $u = (1, 2, 3)$ ,  $v = (2, -3, 1)$  and  $w = (3, 2, -1)$
- i) Find the components of the vector  $u - 3u + 8w$  (2 marks)
- ii) Find the scalars  $c_1, c_2, c_3$  such that  $c_1u + c_2v + c_3w = (6, 14, -2)$  (6 marks)

- b) Let  $\mathbf{u} = (2, -1, 1)$ ,  $\mathbf{v} = (1, 1, 2)$ . Find  $\langle \mathbf{u}, \mathbf{v} \rangle$  and the angle between these two vectors. (3 marks)

#### QUESTION FOUR (20 MARKS)

- a) Given that  $\mathbf{u} = (2, -1, 3)$  and  $\mathbf{w} = (4, -1, 2)$ , find (5 marks)
- $\mathbf{u}_1$ , the projection of  $\mathbf{u}$  onto  $\mathbf{w}$  (3 marks)
  - $\mathbf{u}_2$ , the perpendicular vector to  $\mathbf{w}$
- b) Given that  $\mathbf{u} = (2, -1, 1)$  and  $\mathbf{v} = (1, 1, -1)$ , show that  $\mathbf{u}$  and  $\mathbf{v}$  are orthogonal. (2 marks)
- c) If  $\mathbf{u} = (1, 2, -2)$  and  $\mathbf{v} = (3, 0, 1)$  find the cross product  $\mathbf{u} \times \mathbf{v}$  (5 marks)
- d) Let  $\mathbf{u} = (1, 2, -2)$  and  $\mathbf{v} = (3, 0, 1)$ . Show that  $\langle \mathbf{u}, \mathbf{u} \times \mathbf{v} \rangle$  and  $\langle \mathbf{v}, \mathbf{u} \times \mathbf{v} \rangle = 0$  and hence  $\mathbf{u} \times \mathbf{v}$  is orthogonal to both  $\mathbf{u}$  and  $\mathbf{v}$ . (5 marks)

#### QUESTION FIVE (20 MARKS)

- a) Find the quadratic form of A given that (5 marks)
- $$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$
- b) Show that  $A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$  is a positive matrix (5 marks)
- c) Find the co-ordinates of an arbitrary vector  $(a, b)$  in  $\mathbb{R}^2$  with respect to the basis (10 marks)
- $$S_1 = \{ \mathbf{u}_1 = (1, -2), \mathbf{u}_2 = (3, -4) \}$$
- $$S_2 = \{ \mathbf{v}_1 = (1, 3), \mathbf{v}_2 = (3, 8) \}$$