(Knowledge for Development)

# KIBABII UNIVERSITY

## UNIVERSITY EXAMINATIONS

## 2019/2020 ACADEMIC YEAR

## SECOND YEAR SECOND SEMESTER

## MAIN EXAMINATION

## FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

**COURSE CODE:** **MAP 221/MAT 202**

**COURSE TITLE:** **LINEAR ALGEBRA II**

**DATE:** 05/02/2021 **TIME:** 8 AM -10 AM

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## INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

## QUESTION ONE (30 MARKS)

a) State and prove the **three** properties of a Euclidean space (6 marks)

b) Let u = (2,-1,1) and v = (1,1,2). Find < u,v> and the angle between these vectors. (3 marks)

c) Show that the usual basis of Euclidean space $IR^3$:E={ $e_1$ = (0,1,0), $e_2$ = (1,0,0)and $e_3$ =(0,0,1)} form an orthornormal set in $IR^3$ with the Euclidean inner product. (9 marks)

d) Let F:$IR^3 \rightarrow IR^3$ be defined by F(x,y,z) = ( 2x-3y+4z, 5x-y+2z, 4x+7y). Find the matrix of F relative to the standard basis of $IR^3$ E = { $e_1$ = ( 1,0,0), $e_2$ = (0,1,0), $e_3$ = (0,0,1) } (10 marks)

e) Find the eigen values of the following characteristic of a matrix

$$\begin{vmatrix} \lambda + 2 & 1 \\ -5 & \lambda - 2 \end{vmatrix} = 0$$ (2 marks)

## QUESTION TWO (20 MARKS)

a) Given a vector **v** = ( a, b, c,) in $IR^3$

i) Show that $\cos\alpha = \frac{a}{\|\mathbf{v}\|}$ (2 marks)

ii) Find cos β (2 marks)

iii) Find cos γ (2 marks)

iv) Show that $\frac{\mathbf{v}}{\|\mathbf{v}\|} = (\cos\alpha, \cos\beta, \cos\gamma)$ (2 marks)

v) Show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ (2 marks)

b) Let V be a vector space, **u**∈V and α is a scalar. Prove that the following properties hold.

i) 0**u** = 0 (2 marks)

ii) α0 = 0 (2 marks)

iii) (-1)**u** = -**u** (2 marks)

iv) If α**u** = 0 then α = 0 or **u** = 0 (4 marks)

## QUESTION THREE (20 MARKS)

a) Let **u** = (1, 2, 3), **v** = (2,-3, 1) and **w** = (3, 2,-1)

i) Find the components of the vector **u-3u+8w** (2 marks)

ii) Find the scalars $c_1$, $c_2$, $c_3$ such that $c_1\mathbf{u}+c_2\mathbf{v}+c_3\mathbf{w}$ = (6, 14,-2) (6 marks)

b) Let **u** = (2,-1, 1), **v** = (1, 1, 2). Find< **u**, **v** > and the angle between these two vectors. (3 marks)

## QUESTION FOUR (20 MARKS)

a) Given that **u** = (2,-1,3) and **w** = ( 4,-1,2), find
   - i) $\mathbf{u}_1$, the projection of **u** onto **w** ( 5 marks)
   - ii) $\mathbf{u}_2$, the perpendicular vector to **w** (3 marks)

b) Given that **u** = (2,-1, 1) and **v** = (1, 1,-1), show that **u** and **v** are orthogonal. (2 marks)

c) If **u** = (1,2,-2) and **v** = (3,0,1) find the cross product **u** x **v** (5 marks)

d) Let **u** = (1, 2,-2) and **v** = (3, 0, 1). Show that <**u, u** x **v**> and <**v, u** x **v**> = 0 and hence **u** x **v** is orthogonal to both **u** and **v**. (5 marks)

## QUESTION FIVE (20 MARKS)

a) Find the quadratic form of A given that ( 5 marks)

$$A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$$

b) Show that $A = \begin{bmatrix}4 & 2 & 0\\ 2 & 9 & 0\\ 0 & 0 & 2\end{bmatrix}$ is a positive matrix ( 5 marks)

c) Find the co-ordinates of an arbitrary vector ( a, b) in $\mathbb{R}^2$ with respect to the basis

$s_1 = \{ u_1 = (1,-2), u_2 = (3,-4)\}$ ( 10 marks)

$S_2 = \{v_1 = (1,3), v_2 = ( 3,8)\}$