KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF BSC (PHYSICS)

COURSE CODE: SPH 410

COURSE TITLE: MATHEMATICAL PHYSICS III

DATE: 20/07/2021 **TIME:** 2:00-4:00PM

INSTRUCTIONS TO CANDIDATES

TIME: 2 Hours

Answer question ONE and any TWO of the remaining

KIBU observes ZERO tolerance to examination cheating

Question One

(a) State the associative and closure properties of the elements $\{X, Y, Z, ...\}$ belonging to a group G. (2 marks)

(b) Given the rotation matrix $(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, show that $M(0) = I_2$. (3 marks)

(c) Show that the shortest curve joining two points is a straight line. (4 marks)

(d) Two rings, each of radius a, are placed parallel with their centres $2b$ apart and on a common normal. An open-ended axially symmetric soap film is formed between them. Find the shape assumed by the film is a catenary. (4 marks)

(e) Find the matrix L corresponding to a rotation of co-ordinate axes through an angle θ about the e_3-axis (or x_3-axis). (4 marks)

(f) Evaluate the determinant of the matrix

$$A = \begin{pmatrix} 2 & 1 & -3 \\ 3 & 4 & 0 \\ 1 & -2 & 1 \end{pmatrix} \qquad \text{(4 marks)}$$

(g) A frictionless wire in a vertical plane connects two points A and B, A being higher than B. Let the position of A be fixed at the origin of an xy-coordinate system, but allow B to lie anywhere on the vertical line $x = x_0$. Find the shape of the wire such that a bead placed on it at A will slide under gravity to B in the shortest possible time. (4 marks)

(h) Use tensors to obtain an alternative expression for $\nabla \times (\nabla \times \mathbf{v})$. (4 marks)

Question Two

(a) Define an Abelian group. (2 marks)

(b) Given the set $S = \{1,3,5,7\}$ under multiplication (mod 8):

i) Generate a multiplication table for the group S. (5 marks)

ii) Show that the set S forms a group. (5 marks)

(c) A rotation matrix $M(\theta)$ is defined as $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ where $0 \le \theta \le 2\pi$. Show that $M(\theta)M(\varphi) = M(\theta + \varphi)$. (4 marks)

Question Three

(a) Derive the Euler-Lagrange equation for a string fixed ends. (10 marks)

(b) Find the closed convex curve of length l that encloses the greatest possible area. (10 marks)

Question Four

(a) Which of the following pairs (v_1, v_2) form the components of a first-order Cartesian tensor in two dimensions?

i) $(x_2, -x_1)$
ii) (x_2, x_1)
iii) $(x_1^2, -x_2^2)$ (12 marks)

(b) Show that T_{ij} given by

$$\mathrm{T} = [T_{ij}] = \begin{pmatrix} x_2^2 & -x_1x_2 \\ -x_1x_2 & x_1^2 \end{pmatrix}$$

Are components of a second-order tensor (8 marks)

Question Five

(a) Derive the Euler-Lagrange equation for a fixed lower end and an arbitrary upper end. (10 marks)
(b) Find the shape assumed by a uniform rope when suspended by its ends from two points at equal heights. (10 marks)