(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAA 221/MAT222**

COURSE TITLE: **CALCULUS III**

DATE: 05/02/2021 **TIME: 2 PM -4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Use the 1st principles to determine $\frac{\partial z}{\partial y}$ given that $z = xy^2 - 2yx^2 - 3y^3$ (4 mks)

b) Given that $f(x, y) = xy - 2x$ is the production function of a certain firm. Maximize the this function subject to budget constraint $x + 3y = -4$ (5 mks)

c) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-6)^n (x-3)^n}{n 3^{n+1}}$ (5 mks)

d) Find the volume in the 1st octant between the planes $z = 0$, and $z = x + 2y - 4$ And inside the cylinder $x^2 + y^2 = 36$ (5 mks)

e) An open cylinder has a surface area of $122.34\ cm^2$ Find the radius and the height that will yield maximum volume (6 mks)

f) Verify that the Taylor series expansion for the function $f(x) = cosx$ about $x = 0$ is $cosx = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n}}{(2n)!}$ hence find the Maclaurin series for $f(x) = 2xcosx$ (5 mks)

QUESTION TWO (20 MARKS)

a) If $V = \left\{x, y \ / 0 \leq x \leq 2,\ 0 \leq y \leq \frac{\pi}{2} \ and\ 0 \leq z \leq \frac{\pi}{2}\right\}$ evaluate $\iiint_V 3xcosysinzdV$ (3 mks)

b) Let $z = e^{-3x} secy$ and $x = s^2 t^2 - 4s$ and $y = 2t^3 - s$ find

(i) $\frac{\partial z}{\partial t}$ (4 mks)

(ii) $\frac{\partial z}{\partial s}$ (3 mks)

c) Consider the series $1 + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{7}} + \cdots\cdots\cdots$ using the integral test, determine whether the series converges or diverges (5 mks)

d) Find the volume of the solid bounded by the graphs of $z = 4 - x^2$, $y + z = 2, y = 0$, and $z = 0$ (5 mks)

QUESTION THREE (20 MARKS)

a) Consider the series $\sum_{n=0}^{\infty} \frac{1}{3n}$ use ratio theorem to show that the series converges (3 mks)

b) Find the area of the portion of the cone $x^2 + z^2 = 3y^2$ lying above the yz-plane and inside the cylinder $x^2 + z^2 = 2z$ (6 mks)

c) Given that z is a differentiable function near each (x, y) for $xz^2 + x^2 = 4y - e^{-2y} sin2z$ find

$\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ (6 mks)

d) For what values does the series converge $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$ (5 mks)

QUESTION FOUR (20 MARKS)

a) Let $f(x, y, z) = xtan(yz) - \frac{x^2}{z+x} + 3\ln(xyz)$. Find

(i) f_{xx} (3 mks)

(ii) f_{xyz} (2 mks)

b) Evaluate $\lim_{(x,y)\to(2,2)} \frac{xy-x^2}{\sqrt{x}-\sqrt{y}}$ (4 mks)

c) A particle moves in a circular motion such that its position is given by $x = 2sint$ and $y = cos3t$ for any time t. A force of magnitude $f(x, y) = \frac{2}{3}x^2 - 4y^2 - xy$ is exerted on the particle at a point (x, y). Find an expression for the rate of change of magnitude of the force exerted by the particle with respect to time when $t = 1$ (5 mks)

d) Locate any relative extreme points and determine their nature for the function $f(x, y, z) = x - y^2 - 4xy - 2xz + 5z^2$ (6 mks)

QUESTION FIVE (20 MARKS)

a) Use the Lagrange multipliers to find the local extrema of the function $f(x, y) = x^3 - 2y^2$ Subject to $x^2 + y^2 = 4$ (7 mks)

b) Locate and classify all critical points of $f(x, y, z) = x - 4xy - y^2 + 5z^2 - 2yz$ (7 mks)

c) Investigate the convergence of $\sum_{n=0}^{\infty} \frac{(-1)^{k+1}(k+1)!}{e^{k+1}}$ (6 mks)