(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: **MAP 121**

COURSE TITLE: **ALGEBRAIC STRUCTURES I**

DATE: **20/07/2021** **TIME:** **2 PM - 4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following

i. Group (4marks)

ii. Ring (5marks)

iii. Field (2marks)

b) Define the Klein four group K_4 and proof that it's an abelian group. (8marks)

c) Given the set $S_3 \geq < (123) >$, state the distinct cosets of $< (123) >$ in S_3 (4marks)

d) Use elementary row operations to solve the system, (6marks)

$$x - 2y \pm 3z = 9$$
$$-x + 3y = -4$$
$$2x - 5y + 5z = 17$$

QUESTION TWO (20 MARKS)

a) Define the following

i. Surjective functions (2marks)

ii. Bijective functions (2marks)

iii. Symmetric group (2marks)

b) State and proof the lagranges theorem (6marks)

c) Show that if $|G| = p$ where p is a prime, then G is cyclic (4marks)

d) Let * be an associative binary operation on a set S. Then for all $a \epsilon S$ and all natural numbers m and n, show that

i. $a^m * a^n = a^{m+n}$ (2marks)

ii. $(a^m)^n = a^{mn}$ (2marks)

QUESTION THREE (20 MARKS)

a) Draw the cayley table for the quaternion group (8marks)

b) Show that every cyclic group is abelian (3marks)

c) Generate a 3×3 circulant matrix starting with $[a, b, c]$ (3marks)

d) State 4 examples of fields (4marks)

e) Define a binary operation (2marks)

QUESTION FOUR (20 MARKS)

a) Define the following

i. Simple group (2marks)

ii. Normal subgroup (2marks)

iii. Cyclic group (2marks)

iv. Quotient group (2marks)

v. Index of a group (2marks)

b) Given the set $\mathbb{Z} \geq 3$, state the distinct cosets of $< 3 >$ in $\mathbb{Z}$ (4marks)

c) Find the inverse of the following matrix, whose entries are elements of Z_7 (6marks)

$$A = \begin{bmatrix} 6 & 1 \\ 5 & 3 \end{bmatrix}$$

QUESTION FIVE (20 MARKS)

a) Let $H \leq G$ and $x, y \in G$ then proof that either $xH = yH$ or $xH \cap yH = \emptyset$ (6marks)

b) Give examples of simple groups (3marks)

c) State five examples of binary operations (5marks)

d) State and proof the properties of fields (6marks)