



(Knowledge for Development)

## KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS **2020/2021 ACADEMIC YEAR** 

FIRST YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE **MATHEMATICS** 

COURSE CODE: MAP 121

COURSE TITLE: ALGEBRAIC STRUCTURES I

DATE: 20/07/2021

**TIME**: 2 PM - 4 PM

## **INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

## QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following

i. Group

ii.

iii.

Ring Field

b) Define the Klein four group  $K_4$  and proof that it's an abelian group.

c) Given the set  $S_3 \ge < (123) >$ , state the distinct cosets of < (123) > in  $S_3$  (4marks)

d) Use elementary row operations to solve the system,

QUESTION TWO (20 MARKS)

a) Define the following

Surjective functions i.

Bijective functions ii.

Symmetric group iii.

b) State and proof the lagranges theorem c) Show that if |G| = p where p is a prime, then G is cyclic d) Let \* be an associative binary operation on a set S. Then for all  $a \in S$  and all natural

numbers m and n, show that

i.  $a^m * a^n = a^{m+n}$  $(a^m)^n = a^{mn}$ ii.

QUESTION THREE (20 MARKS)

b) Show that every cyclic group is abelian c) Generate a  $3 \times 3$  circulant matrix starting with [a, b, c]

a) Draw the cayley table for the quaternion group

d) State 4 examples of fields

e) Define a binary operation

 $x - 2y \pm 3z = 9$ -x + 3y = -42x - 5y + 5z = 17

(8marks)

(4marks)

(5marks)

(2marks)

(8marks)

(6marks)

(2marks)

(2marks)

(2marks)

(6marks)

(4marks)

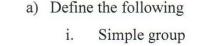
(2marks)

(2marks)

(3marks)

(3marks) (4marks) (2marks)

## **QUESTION FOUR (20 MARKS)**



ii. Normal subgroup iii.

Cyclic group

iv. Quotient group Index of a group

V. Given the set  $\mathbb{Z} \geq 3$ , state the distinct cosets of  $< 3 > \text{in } \mathbb{Z}$ 

Find the inverse of the following matrix, whose entries are elements of  $Z_7$  (6marks)

**QUESTION FIVE (20 MARKS)** 

Let  $H \leq G$  and  $x, y \in G$  then proof that either xH = yH or  $xH \cap yH = \emptyset$ 

Give examples of simple groups

State five examples of binary operations

State and proof the properties of fields

 $A = \begin{bmatrix} 6 & 1 \\ 5 & 3 \end{bmatrix}$ 

(3marks)

(5marks)

(6marks)

(2marks)

(2marks)

(2marks)

(2marks)

(2marks)

(4marks)