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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR
THIRD YEAR FIRST SEMESTER
MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE

COURSE CODE: MAT 321/MAA 311

COURSE TITLE: ODE I

DATE: 20/07/2021

TIME: 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) State the order and degree of the following differential equations

i) $x \left(\frac{dy}{dx} \right)^2 - 4y^3 = e^{2x}$ (2 mks)

ii) $y^{III} + 3(y^{II})^3 = 5y$ (2 mks)

b) Prove that it is homogeneous and solve the differential equation

$$(3x^2 - y^2)dx + (xy - x^3y^{-1})dy = 0 \quad (6 \text{ mks})$$

c) Solve the linear fractional differential equation

$$(2x + y + 6)dx = -(x - 2y - 2)dy \quad (7 \text{ mks})$$

d) Solve the non-homogeneous differential equation

$$2 \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + y = 3\sin x \quad (6 \text{ mks})$$

e) A car engine temperature by the time it is shut off is 180°C . The surrounding air temperature is 35°C . After 15 seconds have elapsed, the engine temperature is 150°C .

(i) How long will it take the engine temperature to cool to 50°C (5 mks)

(ii) Find the engine temperature after 20 seconds (2 mks)

QUESTION TWO [20 MARKS]

(a) Obtain the differential equation having a solution as

$$y = Ae^{-2x} - Be^{3x} + C \quad (7 \text{ mks})$$

(b) Solve the differential equations using appropriate method

(i) $2ydx + x^2dy = 0, y(0) = -2$ (5 mks)

(ii) $(2xy + 3)dx + (x^2 - 1)dy = 0$ (8 mks)

QUESTION THREE [20 MARKS]

- (a) The sum of Kshs 2500 is invested at a rate of 6% per annum compounded continuously.

What will be the amount after 6 years? (4 mks)

- (b) Using the integrating factor solve the differential equation

$$(x^2y - x)dy + (y + 2x^2)dx = 0 \quad (8 \text{ mks})$$

- (c) Solve the following Bernoulli's equation

$$x^2 \frac{dy}{dx} - 2xy = 3y^4; y(1) = 1 \quad (8 \text{ mks})$$

QUESTION FOUR [20 MARKS]

- (a) Verify that the functions $y_1 = x^2 + 2x + 2$ and $y_2 = e^x$ are linearly independent

$$\text{solutions of the differential equation } x \frac{d^2y}{dx^2} - (2 + x) \frac{dy}{dx} + 2y = 0 \quad (7 \text{ mks})$$

- (b) Solve the differential equation $x^2 dy + (y^2 - xy)dx = 0$ (5 mks)

- (c) Use the method of undetermined coefficients to solve

$$2y^{II} - 4y^I - 6y = 3e^{2x} \quad (8 \text{ mks})$$

QUESTION FIVE [20 MARKS]

- (a) Solve the differential equation

$$\frac{d^3y}{dx^3} + 2 \frac{d^2y}{dx^2} - 5 \frac{dy}{dx} - 6y = 0 \quad (6 \text{ mks})$$

- (b) Solve the equation by method of variation of parameter

$$y^{II} + 4y^I + 4y = x^2 e^{-2x} \quad (7 \text{ mks})$$

- (c) Test for exactness and solve the differential equation

$$(6y^2 - x^2 + 3)dy + (3x^2 - 2xy + 2)dx = 0 \quad (7 \text{ mks})$$