KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREES OF BACHELOR OF SCIENCE

(CHEMISTRY)

COURSE CODE: STA 211

COURSE TITLE: PROBABILITY DISTRIBUTION MODELS

DATE: 05/02/2021   **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question One (30 Marks) COMPULSORY

a) Clearly explain 6 application of Chi - Square distribution (6 mks)

b) Use the result $\int_{-\infty}^{\infty} e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{a}$ and $\sqrt{\pi} = \sqrt{\frac{1}{2}}$ and using the

variable change techniques to derive the Chi-Square distribution. (6 mks)

c) Suppose that an average of 30 customers per hour arrive at a shop in accordance with the poisson process. What is the probability that the shopkeeper will wait more than 5 minutes before both of the first two customers arrive? (5 mks)

d) If x and y are discrete random variables and p (x,y) is their joint distribution function given that $f(x, y) = k(x^2 + \frac{xy}{2})$ $0 < x < 1; \ 0 < y < 2$ and if f (x, y) is a pdf,

find the value of k (5 mks)

e) Given that $f(x,y) = e^{-x-y}$ where x > 0, y > 0

i. Show that f is a pdf (4 mks)

ii. Find the marginal distribution of x and y (4 mks)

Question Two (20 Marks)

(a) Compute the moment generating function of a gamma distribution random variable and show that Var $(x) = \alpha\beta^2$ (10 mks)

(b) If $\alpha = 1$, what is $E(x)$ and Var (x) (10 mks)

Question Three (20 Marks)

The pdf of a beta distribution is given by $f(x) = \frac{\Gamma(a+b)}{\Gamma a \Gamma b} x^{a-1}(1-x)^{b-1}$

a) Find the mean of the distribution (10 mks)

b) Find the variance of the beta distribution (10 mks)

Question Four (20 Marks)

(a) Box *I* contains 3 red and 2 blue marbles while Box *II* contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin turns up heads, a marble is chosen from Box *I*; if it turns up tails, a marble is chosen from Box *II*.

i. Find the probability that a red marble is chosen. (5 mks)

ii. Suppose that the one who tosses the coin does not reveal whether it has turned up heads or tails (so that the box from which a marble was chosen is not revealed) but does reveal that a red marble was chosen. What is the probability that Box *I* was chosen (i.e., the coin turned up heads)? (5 mks)

(b) If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random,

(i) 1 bolts will be defective. (3 mks)

(ii) 0 bolts will be defective. (3 mks)

(ii) less than 2, bolts will be defective. (4 mks)

Question Five (20 Marks)

a) Find the moment generating function of a random variable X that is binomially distributed. (5mks)

b) Prove that the mean and variance of a binomially distributed random variable are, respectively, $\mu = np$ and $\sigma^2 = npq$ (5mks)

c) Establish the validity of the Poisson approximation to the binomial distribution. (10mks)