



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2019/2020 ACADEMIC YEAR
SECOND YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE

COURSE CODE: MAT 305

COURSE TITLE: GROUP THEORY I

DATE: 09/02/2021

TIME: 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a) Define the following
- i. an orbit (2marks)
 - ii. transitive (2marks)
 - iii. stabilizer (2marks)
- b) Let $G = S_3$ be the group of permutations of 3 elements, that is $G = \{(1), (12), (13), (23), (123), (132)\}$ and let $H = \{(1), (12)\}$ be a subgroup. Compute the left and right cosets of H . (6 marks)
- c) Let G be a finite group and let H and K be subgroups with relatively prime order. Show that $H \cap K = \{1\}$. (4marks)
- d) Let G be a group. Show that if H is a normal subgroup of order 2, then H belongs to the center of G . (4 marks)
- e) Show that if H is normal in G , then the cosets of H form a group. (5marks)
- f) Show that if H is a subgroup of G , then $|G| = |H|[G : H]$. In particular, if G is finite then $|H|$ divides $|G|$ and $[G : H] = |G|/|H|$. (5 marks)

QUESTION TWO (20MARKS)

- a) Define the following
- i. The order of an element (2 marks)
 - ii. Cyclic group (2marks)
 - iii. Coset (2 marks)

- b) Let the finite group G act on the finite set X , and denote by X^g the set of elements of X that are fixed by g , that is $X^g = \{x \in X, g.x = x\}$. Show that the number of orbits
- $$= \frac{1}{|G|} \sum_{g \in G} |X^g|. \quad (6\text{marks})$$
- c) Let G be a finite group of order n such that all its non-trivial elements have order 2. Show that G is abelian. (4 marks)
- d) Let H be a subgroup of G , and let $g \in G$ but not in H . Show that $H \cup gH$ is a subgroup of G . (4 marks)

QUESTION THREE (20 MARKS)

- a) Define the following
- i. Quotient group (2 marks)
 - ii. The index of a subgroup (2marks)
 - iii. Normal subgroup (2 marks)
- b) Let G be a group. Show that Every normal subgroup of G is the kernel of a homomorphism (5 marks)
- c) Let $f: G \rightarrow H$ be a homomorphism. Show that if K is a subgroup of G , then $f(K)$ is a subgroup of H . If f is an epimorphism and K is normal, then $f(K)$ is normal. (5 marks)
- d) Let $f: G \rightarrow H$ be a homomorphism. Show that If K is a subgroup of H , then $f^{-1}(K) = \{x \in G, f(x) \in K\}$ is a subgroup of G . If K is normal, so is $f^{-1}(K)$. (4marks)

QUESTION FOUR (20 MARKS)

- a) Define the following
- i. Symmetric group (2 marks)
 - ii. Alternating group (2marks)

- b) Show that every group is isomorphic to a group of permutations. (8 marks)
- c) Suppose that a group G acts on a set X . Let $B(x)$ be the orbit of $x \in X$, and let $\text{Stab}(x)$ be the stabilizer of x . Then the size of the orbit is the index of the stabilizer that is $|B(x)| = [G : \text{Stab}(x)]$. If G is finite, then $|B(x)| = |G|/|\text{Stab}(x)|$. In particular, the size of an orbit divides the order of the group. (8 marks)

QUESTION FIVE (30 MARKS)

- a) Define the following
- i. Abelian group (2 marks)
 - ii. Subgroup (2 marks)
 - iii. Group homomorphism (2 marks)
 - iv. Isomorphic groups (2 marks)
- b) Show that $Ha = Hb$ if and only if $ab^{-1} \in H$ for $a, b \in G$. Similarly, $aH = bH$ if and only if $a^{-1}b \in H$ for $a, b \in G$. (6 marks)
- c) Let G be a finite group. Show that if $a \in G$, then $|a|$ divides $|G|$.
In particular, $a^{|G|} = 1$. (3marks)
- d) Let G be a finite group. Show that If G has prime order, then G is cyclic. (3marks)