(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

THIRD YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 307

COURSE TITLE: NUMBER THEORY

DATE: 01/02/2021 **TIME: 2 PM -4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (Compulsory)

a) Find the Remainder when 6^{1987} is divided by 37 (5 marks)

b) Prove that $\log_{10} 7$ is irrational. (5 marks)

c) Find all the solutions in integers to $3456x + 246y = 234$. (5 marks)

d) Show that $\mathbb{Z}_3$ form a group of residues under addition modulo 3. (5 marks)

e) State the five properties of Congruences. (5 marks)

f) State the well-ordering axiomand use it to show that there is no integer between 0 and 1.

(5 marks)

QUESTION TWO

a) Show that the set $A = \{-40,6,7,15,22,35\}$ forms a complete residue set $mod\ 6$ while the set $B = \{-3,-2,-1,1,2,3\}\ mod\ 6$ does not. (10 marks)

b) Show that $n^2 - 23$ is divisible by 24 for infinitely many n . (5 marks)

c) State the Euclidean Algorithm process of finding greatest common divisor and use it to find the greatest common divisor 560 and 600. (5 marks)

QUESTION THREE

a) State and prove the Wilson's theorem. (8 marks)

b) Prove that $n^4 + 4$ is prime only when $n = 1$ for all $n \in \mathbb{N}$ (6 marks)

c) Prove that the product of n consecutive integers is divisible by $n!$ and hence show that $n^5 - 5n^3 + 4n$ *is* always divisible by 120. (6 marks)

QUESTION FOUR

a) State and prove the Euler's theorem. Use the theorem to find the last two digits of 3^{1000} (10 marks)

b) Prove by induction that $\sum_{t=1}^{n-1} t(t+1) = \frac{n(n-1)(n+1)}{3}$,for all natural numbers $n \geq 2$ (5 marks)

c) Solve the congruence $50x \equiv 12 \ mod \ 14$ (5 marks)

QUESTION FIVE

a) Solve the congruence $3x^2 + 3x + 2 = 0 \ (mod\ 10)$. (6 marks)

b) Prove that 7 divides $3^{2n+1} + 2^{n+2}$ for all natural numbers n (5 marks)

c) Find x such that $x \equiv 3 \ mod \ 5$ and $x \equiv 7 \ mod \ 11$ (5 marks)

d) Prove by induction $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ (4 marks)