KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OF BSC (PHYSICS)

COURSE CODE: SPH 411

COURSE TITLE: COMPUTATIONAL TECHNIQUES IN PHYSICS

DATE: 16/07/2021 **TIME:** 2:00-4:00PM

INSTRUCTIONS TO CANDIDATES

TIME: 2 Hours

Answer question ONE and any TWO of the remaining

Section I

a) The height of a right circular cone is increasing at 3 mm/s and its radius is decreasing at 2 mm/s. Determine, correct to 3 significant figures, the rate at which the volume is changing (in cm^3/s) when the height is 3.2 cm and the radius is 1.5 cm. (4 marks)

b) Use the trapezoidal rule to evaluate $\int_0^{2\pi} \frac{1}{1+\sin x} dx$ using 4 intervals. Give the answer correct to 4 significant figures. (4 marks)

c) Evaluate $\begin{vmatrix} (1+j) & j2 \\ -j3 & (1-j4) \end{vmatrix}$ (3 marks)

d) Determine the rate of increase of diagonal AC of the rectangular solid, shown in Figure 1, correct to 2 significant figures, if the sides x, y and z increase at 6 mm/s, 5 mm/s and 4 mm/s when these three sides are 5 cm, 4 cm and 3 cm respectively. (4 marks)

e) Pressure p and volume V of a gas are connected by the equation $pV^{1.4} = k$. Determine the approximate percentage error in k when the pressure is increased by 4% and the volume is decreased by 1.5%. (4 marks)

f) The second moment of area of a rectangle is given by $I = \frac{bl^3}{3}$. If b and l are measured as 40mm and 90mm respectively and the measurement errors are -5mm in b and $+8$mm in l, find the approximate error in the calculated value of I. (4 marks)

g) If $z = f(a, b, c)$ and $z = 2ab - 3b^2c + abc$, find the total differential, dz. (3 marks)

h) Describe importance sampling as a Monte Carlo method. (4 marks)

Section II

Question Two

a) The area A of a triangle is given by $A = 0.5ac \sin B$, where B is the angle between sides a and c. If a is increasing at 0.4 units/s, c is decreasing at 0.8 units/s and B is increasing at 0.2 units/s, find the rate of change of the area of the triangle, correct to 3 significant figures, when a is 3 units, c is 4 units and B is $\pi/6$ radians. (6 marks)

b) Evaluate $\int_0^{2.4} e^{-\frac{x^2}{3}} dx$, correct to 4 significant figures, using the mid-ordinate rule with 6 intervals. (7 marks)

c) Evaluate $\int_0^{\frac{\pi}{3}} \sqrt{\left(1 - \frac{1}{3}\sin^2\theta\right)} d\theta$, correct to 3 decimal places, using Simpson's rule with 6 intervals. (7 marks) [pg. 439]

Question Three

a) Modulus of rigidity $G = \frac{R^4\theta}{L}$, where R is the radius, θ the angle of twist and L the length. Determine the approximate percentage error in G when R is increased by 2%, θ is reduced by 5% and L is increased by 4%. (6 marks)

b) Use the Runge-Kutta method to solve the differential equation:

$$\frac{dy}{dx} = y - x$$

for y_1 and y_2 in the range $0(0.1)0.5$, given the initial conditions that at $x = 0, y = 2$. (14 marks)

Question Four

a) Figure 1 shows a stretched string of length 50 cm which is set oscillating by displacing its mid-point a distance of 2 cm from its rest position and releasing it with zero velocity. Solve the wave equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

where $c^2 = 1$, to determine the resulting motion $u(x, t)$. (10 marks)

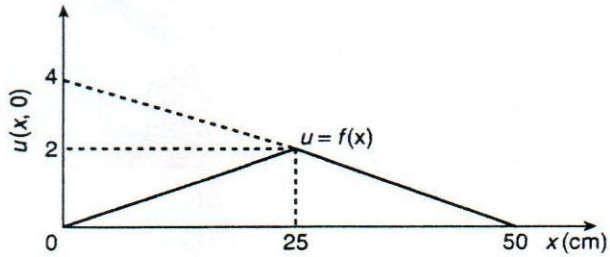

Figure 1

b) A metal bar, insulated along its sides, is 1m long. It is initially at room temperature of 15◦C and at time $t = 0$, the ends are placed into ice at 0◦C. Find an expression for the temperature at a point P at a distance x m from one end at any time t seconds after $t = 0$. (10 marks)

Question Five

1. Determine the inverse of the matrix $\begin{pmatrix} 3 & 4 & -1 \\ 2 & 0 & 7 \\ 1 & -3 & -2 \end{pmatrix}$. (10 marks)
2. A d.c. circuit comprises three closed loops. Applying Kirchhoff's laws to the closed loops gives the following equations for current flow in milliamperes:

$$2I_1 + 3I_2 - 4I_3 = 26$$
$$I_1 - 5I_2 - 3I_3 = -87$$
$$-7I_1 + 2I_2 + 6I_3 = 12$$

Use the Gaussian elimination method to solve for I_1, I_2 and I_3. (10 marks)