(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 423**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATION II**

DATE: 22/07/2021 **TIME:** 9 AM - 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Determine the stability of the system $\dot{x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} x$ (5 marks)

b) Show that there exist a unique solution to the differential equation

$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$, hence find the unique solution. (7 marks)

c) Linearize the system and find the critical points.

$$X' = \begin{bmatrix} x_1^2 - x_2^2 - 1 \\ 2x_2 \end{bmatrix} x$$ (6 marks)

d) solve the following system of differential equations

$$X' = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ (8 marks)

e) State the condition for the following critical points to occur and in each case draw the phase portrait

i) Node . (2 marks)

ii) Saddle point . (2 marks)

QUESTION TWO (20 MARKS)

a) Find the general solution of the system $X' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} x$ (6 marks)

b) Determine the respective fundamental matrix $x(t)$ given that $x(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (11 marks)

c) Hence find $e^{\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} t}$ (3 marks)

QUESTION THREE (20 MARKS)

a) Define a linear system of differential equation. (1 mark)

b) Use matrix method to solve the non-homogenous system of equations

(19 marks)

$$\frac{dx_1}{dt} = x_2 + e^t$$
$$\frac{dx_2}{dt} = 3x_2 - 2x_1 + 1$$

QUESTION FOUR (20 MARKS)

a) Use elimination method to solve the system

(12 marks)

$$\frac{dy}{dt} + 2y + 3x = 2e^t$$
$$\frac{dx}{dt} + 3y + 2x = 0$$

b) Use Picards method to approximate y and z corresponding to x = 0.1 for the particular solution of

$$\frac{dy}{dx} = f(x,y,z) = x + z$$
$$\frac{dz}{dx} = g(x,y,z) = x - y^2$$

Satisfying y=2, z=1 when x=0 .

(8 marks)

QUESTION FIVE (20 MARKS)

c) Define a node of a linear autonomous system. (3 marks)

d) Determine the nature of the critical point (0,0) of the system

$$\frac{dx}{dt} = 2x + 4y$$
$$\frac{dy}{dt} = -2x + 6y$$

And find out whether or not the point is stable. (6 marks)

e) Determine whether or not the solution of the differential equation below is a asymptotically stable or unstable.

$$X' = \begin{pmatrix} -1 & 0 & 0 \\ -2 & -1 & 2 \\ -3 & -2 & -1 \end{pmatrix} x$$ (5 marks)

f) Find the nature of the critical point (0,0) of the non-linear system

$$\frac{dx}{dt} = x + 4y - x^2$$
$$\frac{dy}{dt} = 6x - y + xy$$

(6 marks)