KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2019/2020 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER
SUPPLEMENTARY EXAMINATIONS

FOR THE DEGREE OF BACHELOR OF SCIENCE IN PHYSICS

COURSE CODE: **SPH315**

COURSE TITLE: **MATHEMATICAL PHYSICS II**

DURATION: 2 HOURS

DATE: TIME: 9/2/2021 8 - 10 Am

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other **TWO** (2) Questions.
- Question **ONE** carries **30 MARKS** and the remaining carry **20 MARKS** each.
- ALL Symbols have their usual meaning

QUESTION ONE (30MARKS)

a) Prove that

  i) $\mathcal{L}\{\text{Sin at}\}=\frac{a}{a^2+s^2}$(3marks)

  ii) $\mathcal{L}\{\text{Cosh at}\}= \frac{s}{s^2-a^2}$(3marks)

b) Use the Laplace transform of the first derivative to show that $\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$(4marks)

c) Given $f(z) = 2z^2 - 1$ find $f'(z)$ at $z_o = 1 - i$(3marks)

d) Show that the complex sequence whose n^{th} term is $Z_n = \frac{n^2-2n+3}{3n^2-4} + i\frac{2n-1}{2n+1}$ converges to $\frac{1}{3}+i$ (3marks)

e) Evaluate the integral $\int_C (Z^*)^2 dz$ where C is a straight line joining the points $z = 0$ and $z = 1 + 2i$ (5marks)

f) Find the poles and the corresponding residue of $f(z) = \frac{e^z}{z^2+a^2}$ (5marks)

g) Use the generating function for the Hermite polynomial to find $H_3(x), H_4(x)$ and $H_5(x)$ (4marks)

QUESTION TWO (20 MARKS)

a) Use the calculus of residues to show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$ where $a > b > 0$ (12marks)

b) i) Use the factorial function to evaluate 0! (2marks)

  ii) Define the gamma function $\sqrt{(p)}$ (1mark)

  iii) Verify the recursion relation for gamma functions (5marks)

QUESTION THREE (20 MARKS)

a) Evaluate the integral $\int_C \frac{dz}{(z-z_o)^{n+1}}$ where C is a circle of radius r and centre at z_o and n is an integer. (6marks)

b) State the Cauchy integral formula (2marks)

c) Evaluate $\oint \frac{e^{2z}}{(z+1)^4} dz$ by Cauchy's integral formula where C is any simple closed curve for the cases

  i) C does not enclose $z = -1$ (2marks)

  ii) C encloses the point $z = -1$ (10marks)

QUESTION FOUR (20 MARKS)

a) Determine if the following series converge

  i) $\sum_{n=0}^{\infty}(\log_\pi 2)^n$ (5marks)

  ii) $\sum_{n=0}^{\infty}\frac{4^n+1}{3^n-1}$ (3marks)

b) Find the circle of converge of

  i) $\sum_{n=0}^{\infty} nz^n$ (4marks)

ii) $\sum_{n=0}^{\infty}(z+5i)^{2n}(n+1)^2$ (8marks)

QUESTION FIVE (20 MARKS)

a) Use the gamma function to evaluate $\sqrt{\left(\frac{1}{2}\right)}$ (10marks)

b) By defining the Bessel function show that $J_3(x) = \left(\frac{8}{x^2} - 1\right)J_1(x) - \frac{4}{x}J_0(x)$ (5marks)

c) Use the generating function to list down the first four Laguerre polynomials (5marks)